VOLUME LVII NUMBER 5

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

JUNE 1923

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

June 1923 Vol. LVII, No. 5

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, El Salvador, Argentina, Bolivia, Brazil, Colombia, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Uruguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, and Spain. ¶Postage is charged extra as follows: for Canada, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12*s*. 6*d*. each; single copies, including postage, 4*s*. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.65 each; single copies, including postage, Yen 1.68 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the Act of March 3, 1879.

Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LVII — JUNE 1923 — NUMBER 5

THE CROSSED-ORBIT MODEL OF HELIUM, ITS IONIZATION POTENTIAL, AND THE LYMAN SERIES

By LUDWIK SILBERSTEIN

ABSTRACT

The energy formula for crossed-orbit model of the normal helium atom.—From simple dynamical considerations it is shown that the energy of the model, with the two orbits assumed as rigorously circular, is given by the formula, $E=-7N_{\infty}ch\left[1-\frac{1}{4\pi}F\left(\frac{i}{2}\right)\right]$, where i is the mutual inclination of the planes of the two orbits and F is the complete elliptic integral of the first kind. From this general formula it is found that for Bohr's model, in which $i=120°$, the *ionizing potential* is 24.35 volts, as compared with 24.5 volts recently obtained by Lyman. By means of the *formula for the wave-lengths* of the emitted radiation, $\nu=N\left[3-\frac{7}{4\pi}F\left(\frac{i}{2}\right)\right]$, it is found that the simple rational values, $-\cos i=\frac{5}{9}, \frac{3}{8}, \frac{2}{9}, \frac{1}{8}$, correspond to the lines, $\lambda_1=584.4$, $\lambda_2=537.1$, $\lambda_3=522.3$, and $\lambda_4=515.7$, of *the Lyman series*. The author disclaims any responsibility for the dynamical legitimacy of the model. In an appended note two more lines are shown to be covered by $-\cos i=\frac{5}{6}$ and $\frac{7}{13}$, and a regularity of the whole array of fractions is pointed out.

The model of the normal helium atom proposed by Bohr in his Fysisk Forening lecture[1] consists of two electrons describing around the nucleus two equal one-quantic orbits (1_1) which are quasi-circular and whose planes are mutually inclined at $i=120°$, and are themselves spinning "slowly" around the permanent axis of angular momentum of the whole system. Bohr states (p. 33) that the

[1] Translated in *Zeitschrift für Physik*, **9**, 1–67, 1922.

analysis of the said configuration, conducted with the aid of Kramers, requires a large amount of computation which had thus far led to no conclusive results, although it seems promising with regard to a correct ionization potential. More than a year has now elapsed without any further announcements to that effect being published by Bohr or Kramers. In the meantime, J. H. van Vleck has announced in a note to his paper[1] that "calculation has given an ionization potential of 20.7 volts," for the aforesaid model, that is. Yet, the details of computation leading to this disappointing potential value not being very transparent, one did not feel ultimately convinced as to the fate of this latest model, a successor to three or four luckless ones. Under these circumstances it has seemed worth while to attempt an independent evaluation by a straightforward method which recently suggested itself. Its publication in the present paper seems the more justified, as the result arrived at was surprisingly close to Lyman's latest estimate, which, as I am informed by Professor Lyman, is one volt lower than that usually quoted, and amounts ultimately to 24.5 volts.

It will be expressly understood, however, that the dynamical possibility of the model, as satisfying permanently and with sufficient accuracy the equations of motion, will be entirely left to the judgment of others. In fine, the legitimacy of the model itself, with its sufficiently "quasi-circular" orbits, being taken for granted, I propose merely to compute its energy (with the usual Coulomb law of interaction between all the three bodies), and thence the required ionization potential.

By a well-known theorem, due to Burgers, the average kinetic energy of the system is equal to minus one-half its average potential energy, and since the total energy, E, remains constant, we have

$$E=\bar{E}=\tfrac{1}{2}\bar{E}_{pot}\,,$$

and, if a be the radius of either orbit, and $\rho=1/r$ the reciprocal of the mutual distance of the electrons at any instant,

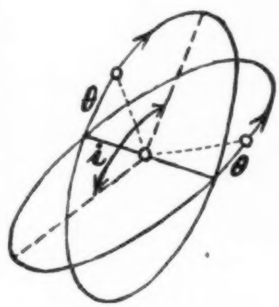
$$E=-\frac{e^2}{a}[2-\tfrac{1}{2}a\bar{\rho}]\,.$$

[1] *Philosophical Magazine*, **44**, 869, 1922.

The configuration being symmetrical by assumption, the two trabants will pass the nodes simultaneously, being then on the opposite ends of a diameter. Thus, if the "slow" spinning of the orbits themselves (as announced by Bohr, *loc. cit.*) is slow enough compared with the orbital angular velocity of the electrons, the approximate value of the radius, a, can be determined by the same reasoning as for Bohr's older ring electron (which gave a much too high potential), which, in usual symbols, gives $e^2/a=\frac{7}{2}chN_\infty$. Consequently,

$$E=-7N_\infty ch(1-\tfrac{1}{4}a\bar{\rho}). \tag{1}$$

It remains to find $\bar{\rho}$. Now if the mutual inclination, i, of the orbit planes be defined as explained in the figure, and if the azimuth angles, $\theta=\bar{\omega}t$, be counted from the nodes, we have, by elementary geometry, at any instant t,



$$r^2/a^2=4\cos^2\theta+2(1+\cos i)\sin^2\theta,$$

whence, the required average,

$$\bar{\rho}=\frac{1}{4\pi a}\int_0^{2\pi}\frac{d\theta}{\sqrt{1-\frac{1}{2}(1-\cos i)\sin^2\theta}},$$

or $\pi a\bar{\rho}=F(k)$, where F is the complete elliptic integral of the first kind, modulo

$$k=\sqrt{\frac{1-\cos i}{2}}.$$

If we put, as usual, $k=\sin\alpha$, then $\alpha=\frac{1}{2}i$, the semi-inclination. Thus, writing now α instead of the modulus itself, as is customary in elliptic tables, we have for the average reciprocal distance of the two trabants,

$$\bar{\rho}=\frac{1}{\pi a}F\left(\frac{i}{2}\right), \tag{2}$$

a handy kinematical formula which may be interesting on its own account.

Substituting this into (1), and leaving, for the present, i unspecified, we have for the negative energy of the system, or of what will hereafter be referred to as the i-model,

$$-E=7N_{\infty}ch\left[1-\frac{1}{4\pi}F\left(\frac{i}{2}\right)\right]. \tag{3}$$

Next, drag away one electron to "infinity," allowing the other to settle itself in a one-quantic orbit. Since the negative energy of this residue, the ionized atom, He^{+}, is simply $4Nch$, where $N=N_{\infty}:(1+m/M)$, and since the small difference $N_{\infty}-N$ is irrelevant for the purpose in hand, we have for the ionization work,

$$W=Nch\left[3-\frac{7}{4\pi}F\left(\frac{i}{2}\right)\right], \qquad N \doteq 1.0973\cdot 10^{5},$$

or, for the equivalent wave-number (of the flash emitted at the return of the vagabond),

$$\nu=N\left[3-\frac{7}{4\pi}F\left(\frac{i}{2}\right)\right]. \tag{4}$$

Now, for Bohr's model, $i=120°$, and, by a four-figure table, $F(60°)=2.1565$. Thus $\nu=1.7987N$, and since N is equivalent to 13.54 volts, the corresponding ionization potential amounts to

$$V=24.35 \text{ volts}, \tag{5}$$

which is remarkably close to 24.5, the latest observed value. The corresponding wave-length, or the limit of Lyman's (and Fricke's) extremely ultra-violet series, would be $\lambda_{\infty}=506.6$ A.

Whether this is a mere "chance" coincidence, or has some "deeper significance" (whatever that means), I am unable to say. Still less, whether this model of normal helium is a dynamically legitimate one. This, as stated before, will be left entirely to those who have proposed it.

Thus far the particular case of the Bohr model. Now, by way of mere curiosity, suppose, for the moment, that there are dynamically possible (and stable) states of the i-model also for some inclinations other than 120°. Then its energy will be as in (3),

and the wave-number of radiation emitted at the passage from He^+ to this i-model will be given by (4). Bohr's model corresponds to $\cos\, i = -\frac{1}{2}$. He supports this choice only by a terse appeal (p. 32) to the quantizing principle of angular momentum.

At any rate, it has seemed interesting to apply formula (4), even regardless of its significance or deduction, to some other simple rational values of $\cos\, i$, especially with a view of covering, perhaps, some of the four observed members of Lyman's series, $oS - mP$, which are $\lambda_1 = 584.4$, $\lambda_2 = 537.1$, $\lambda_3 = 522.3$, $\lambda_4 = 515.7$, with the aforesaid λ_∞ as limit.

The region beyond 500 A being thus far barren or unexplored, values of $\cos\, i > -\frac{1}{2}$ are without interest, and thus the next simple ones worth considering are $-\frac{2}{3}$, $-\frac{3}{4}$, and so on. The results obtained on this somewhat adventurous quest were as follows:

$$\cos\, i = -\tfrac{2}{3}, \quad \frac{i}{2} = 65^\circ\!.905, \quad F = 2.340_4, \quad \text{gave} \quad \lambda = 537.2,$$

remarkably close to the observed λ_2; the next tried, $\cos\, i = -\frac{3}{4}$, gave $\lambda = 561.9$, which is without interest; but the very next trial,

$$\cos\, i = -\tfrac{4}{5}, \quad \frac{i}{2} = 71^\circ\!.565, \quad F = 2.578_1, \quad \text{gave} \quad \lambda = 582.7,$$

which roughly corresponds to Lyman's first line, and

$$\cos\, i = -\tfrac{3}{5}, \quad \frac{i}{2} = 63^\circ\!.435, \quad F = 2.257_3, \quad \text{yielded} \quad \lambda = 522\ 9,$$

which is close enough to the observed λ_3. But one more member, 515.7, observed by Lyman, remained uncovered. Working back from this, the required F will be found, by (4), to be 2.2131, and the corresponding semi-inclination, $61^\circ\!.97$, whence $-\cos\, i = 0.558$, whereas the nearest simple fraction, $\frac{5}{9}$, is 0.555..... But whether 5 and 9 are still "simple" integers must be left to everyone's own judgment. Taking $\frac{5}{9}$, we have $i/2 = 61^\circ\!.870$, $F = 2.2094$, and $\lambda = 515.1$. In fine, formula (4) gives the correct ionization potential (5) for $-\cos\, i = \frac{1}{2}$, and, as a curious addition, the lines

$$\lambda_4, \quad \lambda_3, \quad \lambda_2, \quad \lambda_1,$$

respectively, for $-\cos i = \frac{5}{9}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}$, with the deviations -1.7, $+0.6$, $+0.1$, and $+0.6$ angstroms, the experimental error limits being ± 0.2 A.

Whether or not these additional states of the model are dynamically and otherwise legitimate could be decided only by a thorough analysis which the writer is not in the position to offer.

ROCHESTER, N.Y.
March 3, 1923

NOTE ADDED MAY 12, 1923

Since the above was printed, the long awaited investigation of Dr. Kramers (*Zeitschr. für Physik*, **13**, 312–341, 1923) has reached America. The net result of this investigation, in which the computation of the mutual perturbation of the electrons is pushed to the second approximation, is that the Bohr model gives a much too low ionization potential, namely 20.7 volts (identical with van Vleck's previous result), and that the contemplated quantized motion is not even stable in the mechanical sense of the word. Kramers, however, who speaks also in the name of Bohr (*loc. cit.*, pp. 339, 340), does not at all seem to be discouraged by this unexpected "negative result" of his laborious computations. On the contrary, after a consultation with Dr. Bohr, he sees in this failure only a proof that "already in this simple case [of two trabants only] ordinary mechanics loses its validity," and does not swerve from his belief in "the correctness" of the model itself. In fine, Kramers and Bohr are unanimous in condemning, not the model, but "ordinary" mechanics. As yet, however, they do not see their way for proposing any definite modification or cutting of classical mechanics to make it applicable to atomic systems.

Under these circumstances, the alternative method of computation given above seems the more interesting. Originally dictated by the writer's ignorance of details, this method now appears brutally simple and "classically" wrong. For it amounts to adding, in (1) or (3), to the quantized central energy the averaged perturbation function—a theorem known to be valid only when the latter is a small fraction of the former. But it is precisely such a manifest infringement of the classical laws yielding fairly correct results, which may serve as a hint how to modify these mechanical laws for intra-atomic purposes. This is the reason why the foregoing treatment is here left unchanged.

In the second place, it may be well to add the following peculiarities of the spectrum formula (4) as such, noticed in the meantime.

If the simple rational values of $-\cos i$ are written out orderly, descending in magnitude,

$$\frac{4}{5}\left(\frac{3}{4}\right)\frac{2}{3}\left(\frac{5}{8}\right)\frac{3}{5}\left(\frac{4}{7}\right)\frac{5}{9},$$

every second (bracketed) covers no observed line, while the remaining ones represent orderly the first four members of the Lyman series $0S-mP$. Now, extrapolating this intermittence and continuing the regular sequence of the last three fractions by

$$\left(\frac{6}{11}\right)\ \frac{7}{13},$$

one would expect the former to cover no line, and the latter to cover the line $0S-5P$ which, though not yet observed, can confidently be expected. Now, the wave-length of this line, with Lyman's $0S$ and the usual $5P$, would be $\lambda_5=512.1$, while our formula (4) gives, for $\cos i=-7/13$,

$$\lambda=512.3.$$

Turning to the left hand of the above sequence, the next fraction $\frac{5}{6}$ has naturally seemed worth trying. To this value of $-\cos i$ corresponds $i/2=73^\circ.221$, $F=2.6642$ and, by (4),

$$\lambda=601.2$$

which is very close to the "single line at 600.5 ± 0.3" repeatedly observed by Lyman. Moreover, the combination line

$$0S-1S=198{,}300-32{,}033$$

would lie at $\lambda=601.3$, which is still closer to our result.

Gathering the scattered results we have the following correlation (in which bracketed fractions cover no lines):

$$\begin{array}{c|cccccccccccc}
\frac{5}{6} & \frac{4}{5} & \left[\frac{3}{4}\right] & \frac{2}{3} & \left[\frac{5}{8}\right] & \frac{3}{5} & \left[\frac{4}{7}\right] & \frac{5}{9} & \left[\frac{6}{11}\right] & \frac{7}{13} & \cdot & \cdot & \cdot\ \frac{1}{2} \\
0S-1S & \lambda_1 & & \lambda_2 & & \lambda_3 & & \lambda_4 & & \lambda_5 & & & \lambda_\infty
\end{array}$$

The regular intermittency, as far as $0S-mP$ is concerned, is manifest. The position of $0S-1S$, the "queer" line (Compton), is correspondingly queer. Yet even this fits into the further regularity of the whole sequence of fractions, pointed out to me by my friend, Professor A. S. Eve, of Montreal, to wit, that the differences between the successive fractions are all of the form $\frac{1}{np}$, thus $5\cdot5-4\cdot6=1$, $4\cdot4-3\cdot5=1$, and so on.

The substance of this note was given in a paper read at the Washington meeting of the Physical Society, April 21, and the whole subject was expounded in a lecture delivered two days later at the Bureau of Standards, followed by a discussion.

INVESTIGATIONS ON PROPER MOTION

TENTH PAPER: INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 33, N.G.C. 598[1]

By ADRIAAN VAN MAANEN

ABSTRACT

Measures of internal motion in the spiral nebula M 33 (N.G.C. 598).—Comparison of two photographs taken in 1910 and 1922 by Ritchey and Humason, respectively, gives, with respect to twenty-four comparison stars, the annual proper motion of the nebula, $\mu_\alpha = +0''.003$, $\mu_\delta = -0''.004$, and the motions of 399 nebular points freed from this motion. The internal motions are shown on Plate XIX. They can be interpreted as a rotation or as a motion outward along the arms of the spiral, preferably the latter. Taken as a rotation, the motions indicate periods from 60,000 to 240,000 years.

Reality of measured displacements in spirals.—A summary of the results for seven spirals, M 33, 51, 63, 81, 94, 101, and N.G.C. 2403, shows that the displacements found cannot have been caused by (*a*) the telescope, (*b*) the quality of the plates, (*c*) the measuring instrument, (*d*) the measurer. Apparently they must be accepted as representing actual internal motion. As such they are in agreement with the theory of cosmogony lately proposed by Jeans.

Parallaxes of the larger spiral nebulae.—These seem to lie between a few ten-thousandths and a few thousandths of a second of arc. The corresponding diameters range from several light-years to several hundred light-years. The larger spirals are therefore enormous as compared with our solar system, but small in comparison with the system of the Milky Way.

In 1921 a preliminary note on the internal motion in this nebula was published in the *Proceedings of the National Academy of Sciences;*[2] it included measures of thirty nebulous points on two plates taken at the 25-foot focus of the 60-inch reflector and of twenty-two points on the photographs taken at the 80-foot focus of the same instrument. The measures were all made with the monocular arrangement of the Zeiss stereocomparator, while those planned for several hundred nebular points on the first pair of plates were postponed until the new stereocomparator, then under construction in the instrument shop of the observatory, should be completed. Before this was finished, the plate taken by Mr. Duncan in 1920 was accidentally broken by him and it was necessary to wait until a new exposure could be made. This was secured by Mr. Humason on September

[1] *Contributions from the Mount Wilson Observatory*, No. 260.

[2] *Mt. Wilson Comm.*, No. 71.

23 and 24, 1922. As the older plate taken by Mr. Ritchey dates from August 5, 6, and 7, 1910, the interval is 12.133 years.

Twenty-four comparison stars and 400 points, presumably belonging to the nebula, were selected for measurement. The measures and reductions were carried out in the same way as for other spirals studied.

The measures were made in four positions, east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. The measures in right ascension were combined into one set, and those in declination into another; then the measured quantities were multiplied by 0.688 to reduce the values expressed in parts of the micrometer screw to annual motions in thousandths of a second of arc. These quantities, m_α and m_δ, respectively, were used as the first members in equations of condition of the form:

$$\left.\begin{aligned} m_\alpha &= a+bx+cy+dx^2+exy+fy^2+\mu_\alpha \\ m_\delta &= a'+b'x+c'y+d'x^2+e'xy+f'y^2+\mu_\delta \end{aligned}\right\} \qquad (1)$$

in which $a \ldots . f$, $a' \ldots . f'$, are the plate constants, x and y the co-ordinates in right ascension and declination, and μ_α and μ_δ the annual proper motions. By a least-squares solution the plate constants were determined from two sets of equations of the form (1), yielded by the twenty-four comparison stars. These constants were substituted into equations of the form (1) for all objects measured, thus giving for both stars and nebular points μ_α and μ_δ, the components in right ascension and declination of the motions with respect to the mean of the comparison stars.

For the comparison stars these quantities are given in the fourth and fifth columns of Table I; the second and third columns give the positions with respect to the center of the nebula, accurate to a tenth of a minute of arc.

In order to derive the internal motions of the nebula, the values μ_α and μ_δ of the nebular points must be freed from the motion of the nebula as a whole. One point, No. 367, shows so large a motion ($\mu = 0\overset{''}{.}136$) that it cannot be a part of the nebula, and is accordingly omitted in the following discussion.

a) The mean motion of the 399 remaining points is

$$\mu_\alpha = +0''.002; \qquad \mu_\delta = -0''.003^5.$$

b) Combining the mean motion in quadrants I and III, we have

$$\mu_\alpha = +0''.004^5; \qquad \mu_\delta = -0''.003^5$$

while for quadrants II and IV

$$\mu_\alpha = +0''.003^5; \qquad \mu_\delta = -0''.005^5.$$

TABLE I

CO-ORDINATES AND ANNUAL MOTIONS OF THE COMPARISON STARS

No.	x	y	μ_α	μ_δ
a	−1'.6	−4'.5	−0".003	0".000
b	−2.9	−2.7	−3	−2
c	−2.7	−11.0	−11	+8
d	−8.0	−11.4	+5	−7
e	−7.0	−5.7	−2	−8
f	−13.2	−1.4	+2	+8
g	−11.9	+2.4	+6	−7
h	−8.6	+2.4	−4	+3
i	−7.6	+3.7	−7	+3
j	−1.4	+1.7	+7	0
k	−3.3	+6.4	−2	+6
l	−2.1	+11.4	−2	+1
m	+0.5	+10.3	+4	−2
n	+5.2	+8.9	+3	−5
o	+11.6	+4.0	−5	+4
p	+4.1	+4.0	0	+6
q	+7.0	+2.1	0	+8
r	+1.3	+3.2	−4	−15
s	+8.0	−1.1	+4	−10
t	+10.7	−6.8	−4	−4
u	+6.4	−9.2	+1	+1
v	+1.7	−10.3	0	+4
w	+3.3	−4.0	+13	+1
x	+0.5	−2.5	+0.002	+0.008

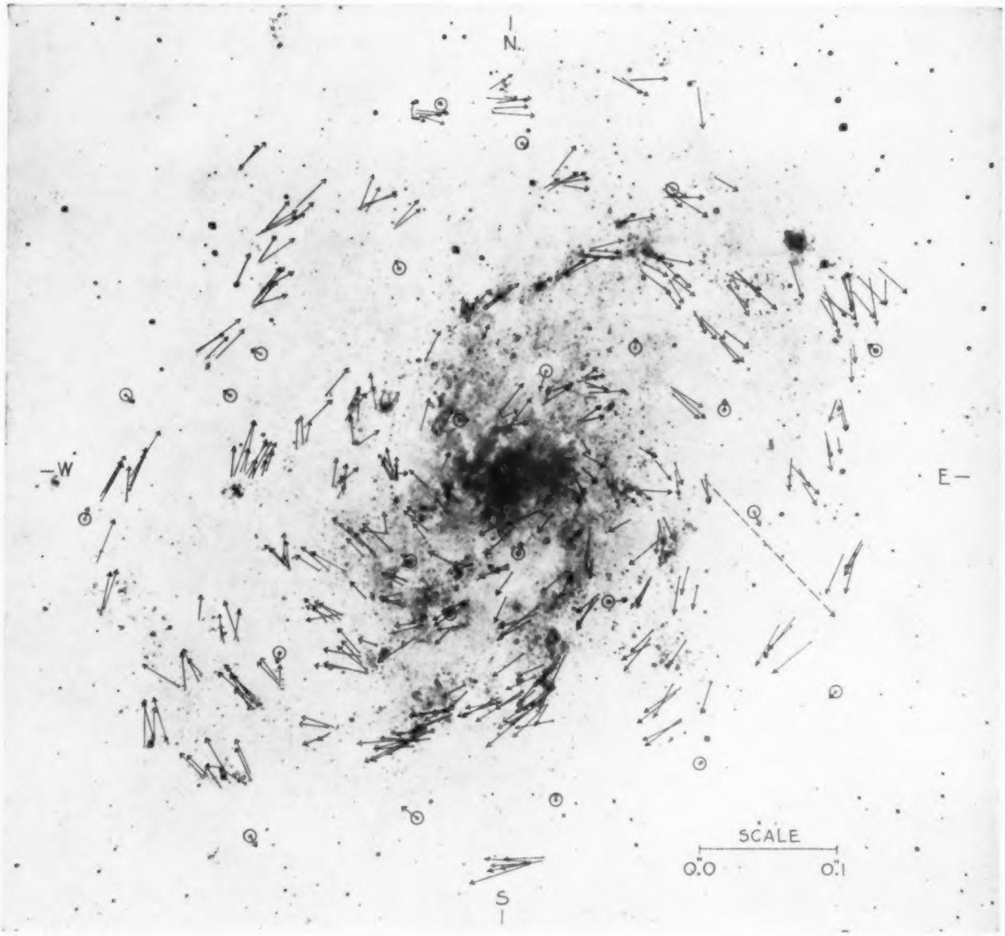
All four quadrants combined give

$$\mu_\alpha = +0''.004; \qquad \mu_\delta = -0''.004^5.$$

c) Using only the 293 points, within 10′ from the center of the nebula, which have a more symmetrical distribution, we find for quadrants I and III

$$\mu_\alpha = -0''.004; \qquad \mu_\delta = -0''.003^5$$

PLATE XIX



INTERNAL MOTIONS IN MESSIER 33

The arrows indicate the directions and magnitudes of the annual motions. Their scale (0.″1) is indicated on the illustration. The scale of the nebula is 1 mm = 12.″8. The comparison stars are inclosed in circles.

while for quadrants II and IV

$$\mu_\alpha = +0''.003; \qquad \mu_\delta = -0''.004^5.$$

For all quadrants combined, we find

$$\mu_\alpha = +0''.003^5; \qquad \mu_\delta = -0''.004.$$

For the motion of the nebula as a whole, the mean result from the three methods is

$$\mu_\alpha = +0''.003, \qquad \mu_\delta = -0''.004.$$

Subtracting these values from the annual motions, μ_α and μ_δ, of the individual points, we derive the internal motions, which are given in the fourth and fifth columns of Table II. These motions are plotted in Plate XIX; for the comparison stars, which are surrounded by circles, the motions of Table I are used; the motion of point 367 is indicated by a broken line. The scale of the motions is indicated in the lower right-hand corner. The length of the arrows represents the motions during an interval of about 2,500 years.

The second and third columns of Table II give the co-ordinates of the points measured, with respect to the center of the nebula, to tenths of a minute of arc. In a few cases, points close together thus appear with identical co-ordinates. This is of no consequence, because, if necessary, these cases can be distinguished by their motions.

The internal motions are resolved into: (*a*) rotational and radial components; (*b*) components along and at right angles to the spiral arms (stream and transverse components).

The plane of the nebula is probably inclined a little to the tangential plane of the celestial sphere, but, as the axes are in the ratio of about 1 to 1.25, the components will in no case be affected by more than 25 per cent, or in the mean by about 10 per cent.

The results are given in the sixth, seventh, eighth, and ninth columns of Table II, where the positive sign is used for motions in the direction N-E-S-W, and outward.

TABLE II

CO-ORDINATES AND ANNUAL INTERNAL MOTION

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
1	−12′.7	− 4′.5	+0″.007	+0″.031	+0″.027	−0″.017	+0″.026	−0″.019
2	−12.7	− 4.3	+ 9	+ 28	+ 23	− 18	+ 21	− 20
3	−12.9	− 2.9	+ 13	+ 32	+ 28	− 20	+ 29	− 19
4	−12.7	− 1.0	+ 15	+ 27	+ 26	− 17	+ 28	− 12
5	−12.7	− 0.8	+ 16	+ 27	+ 26	− 17	+ 28	− 13
6	−12.6	− 0.6	+ 13	+ 26	+ 25	− 14	+ 27	− 10
7	−11.9	− 1.0	+ 2	+ 25	+ 25	− 3	+ 25	− 1
8	−11.8	− 0.5	+ 11	+ 25	+ 25	− 10	+ 26	− 6
9	−11.8	− 0.3	+ 17	+ 27	+ 29	− 13	+ 31	− 7
10	−11.8	− 0.2	+ 22	+ 33	+ 33	− 22	+ 38	− 11
11	−11.4	− 8.7	0	+ 35	+ 28	− 21	+ 23	− 27
12	−11.1	− 8.4	− 7	+ 25	+ 24	− 9	+ 23	− 12
13	−11.1	− 8.6	0	+ 24	+ 19	− 14	+ 18	− 17
14	−10.7	− 8.6	− 11	+ 33	+ 32	− 13	+ 30	− 18
15	− 9.5	− 9.5	− 15	+ 16	+ 22	0	+ 21	− 4
16	− 9.4	− 9.5	− 14	+ 14	+ 20	0	+ 19	− 5
17	− 8.9	− 9.4	− 10	+ 1	+ 8	+ 6	+ 10	+ 3
18	− 8.7	− 9.4	− 14	+ 24	+ 26	− 9	+ 22	− 16
19	− 8.9	−10.0	− 11	+ 15	+ 18	− 5	+ 16	− 10
20	− 8.1	− 9.9	− 20	+ 11	+ 22	+ 4	+ 23	− 3
21	− 8.0	− 9.7	− 11	+ 23	+ 23	− 12	+ 18	− 18
22	− 8.0	− 9.5	− 11	+ 24	+ 23	− 12	+ 20	− 17
23	−10.2	− 6.7	− 27	+ 15	+ 28	+ 13	+ 30	+ 6
24	−10.0	− 6.8	− 2	+ 28	+ 26	− 13	+ 20	− 19
25	− 9.7	− 6.5	− 11	+ 23	+ 25	− 5	+ 24	− 10
26	− 9.5	− 6.4	− 9	+ 11	+ 14	+ 2	+ 14	− 1
27	− 9.5	− 4.6	+ 2	+ 18	+ 15	− 10	+ 13	− 12
28	− 8.9	− 5.2	− 1	+ 19	+ 17	− 9	+ 12	− 14
29	− 8.7	− 5.4	− 2	+ 14	+ 13	− 5	+ 11	− 9
30	− 8.4	− 5.2	+ 3	+ 22	+ 16	− 15	+ 10	− 19
31	− 8.3	− 5.2	− 11	+ 29	+ 30	− 6	+ 27	− 14
32	− 8.1	− 6.8	− 14	+ 17	+ 22	− 2	+ 20	− 10
33	− 8.1	− 7.0	− 15	+ 21	+ 26	− 3	+ 24	− 11
34	− 8.0	− 7.3	− 24	+ 22	+ 32	+ 3	+ 32	− 5
35	− 7.8	− 7.0	− 17	+ 16	+ 23	+ 3	+ 22	− 6
36	− 8.1	− 7.5	− 12	+ 26	+ 28	− 7	+ 24	− 16
37	− 7.8	− 7.5	− 14	+ 14	+ 19	+ 1	+ 19	− 5
38	− 7.8	− 7.3	− 16	+ 22	+ 27	− 4	+ 25	− 11
39	− 7.0	− 6.7	− 16	+ 17	+ 23	0	+ 23	− 7
40	− 7.0	− 6.7	0	+ 13	+ 9	− 9	+ 6	− 11
41	− 7.2	− 6.5	− 12	+ 15	+ 19	− 2	+ 18	− 7
42	− 6.7	− 3.0	− 6	+ 12	+ 13	+ 1	+ 13	− 2
43	− 6.8	− 3.0	− 2	+ 22	+ 21	− 8	+ 19	− 11
44	− 6.8	− 3.0	− 13	+ 15	+ 19	+ 6	+ 19	+ 3
45	− 6.8	− 2.9	− 15	+ 14	+ 19	+ 8	+ 20	+ 5
46	− 7.0	− 2.9	+ 1	+ 24	+ 21	− 10	+ 21	− 11
47	− 8.6	− 0.2	+ 2	+ 22	+ 22	− 2	+ 22	+ 2
48	− 8.6	0.0	+ 15	+ 40	+ 40	− 15	+ 41	− 12
49	− 8.3	0.0	+ 6	+ 17	+ 17	− 6	+ 17	− 3
50	− 8.1	0.0	+ 7	+ 37	+ 37	− 7	+ 38	0
51	− 8.1	0.0	+ 16	+ 24	+ 25	− 15	+ 27	− 10
52	− 8.1	− 0.2	+ 13	+ 28	+ 27	− 13	+ 28	− 9
53	− 8.0	− 0.2	+ 2	+ 13	+ 13	− 2	+ 13	− 1
54	− 7.6	0.0	+ 4	+ 16	+ 16	− 4	+ 16	− 3
55	− 7.5	+ 0.2	+ 5	+ 15	+ 15	− 6	+ 16	− 3
56	− 7.6	+ 0.2	+ 11	+ 19	+ 19	− 11	+ 20	− 9
57	− 8.0	0.0	+ 12	+ 23	+ 22	− 13	+ 23	− 10
58	− 9.1	+ 3.7	+ 20	+ 20	+ 26	− 12	+ 27	− 9
59	− 9.4	+ 3.5	+ 17	+ 16	+ 21	− 11	+ 22	− 7
60	− 9.7	+ 3.8	+ 37	+ 24	+ 38	− 24	+ 41	− 15
61	− 9.7	+ 3.8	+ 21	+ 12	+ 19	− 15	+ 21	− 11
62	− 7.6	+ 5.2	+ 11	+ 18	+ 21	0	+ 21	+ 3
63	− 7.8	+ 5.2	+ 17	+ 20	+ 25	− 4	+ 25	− 5
64	− 8.0	+ 5.2	+ 26	+ 8	+ 20	− 18	+ 20	− 18
65	− 8.0	+ 5.2	+ 20	+ 26	+ 33	− 2	+ 33	− 2
66	− 7.8	+ 5.4	+ 21	+ 24	+ 32	− 4	+ 32	− 5
67	− 8.4	+ 5.9	+ 11	+ 23	+ 24	+ 5	+ 24	+ 6
68	− 7.6	+ 5.7	+ 21	+ 11	+ 21	− 10	+ 21	− 11
69	− 7.8	+ 6.7	+ 12	+ 19	+ 22	+ 2	+ 22	+ 3
70	− 7.6	+ 6.5	+ 24	+ 15	+ 27	− 7	+ 27	− 9

TABLE II—*Continued*

No.	x	y	μ_{α}	μ_{δ}	Rotational	Radial	Stream	Transverse
71	−7′.8	+7′.5	+0″.029	+0″.014	+0″.031	−0″.011	+0″.031	−0″.010
72	−7.6	+7.5	+26	+15	+29	−7	+29	−6
73	−7.5	+7.6	+16	+19	+25	+3	+25	+4
74	−7.0	+7.8	+15	+9	+18	−4	+18	−4
75	−6.8	+7.8	+18	+15	+23	+1	+23	0
76	−6.7	+8.1	+22	+21	+30	+3	+30	+3
77	−8.3	+9.4	+15	+17	+22	+4	+22	+2
78	−8.3	+9.5	+17	+17	+24	+2	+24	−1
79	−4.6	+8.3	+10	+25	+21	+18	+23	+13
80	−4.5	+8.1	+9	+17	+16	+11	+17	+7
81	−4.5	+8.3	+24	+10	+26	−1	+25	−6
82	−4.5	+8.4	+23	+6	+23	−6	+22	−10
83	−5.6	+2.4	+16	+20	+24	−8	+24	−8
84	−6.2	+1.6	+17	+17	+20	−13	+20	−13
85	−6.2	+1.0	+2	+15	+15	−1	+15	−2
86	−6.4	+1.0	−1	+21	+20	+5	+21	+3
87	−6.5	+0.8	+1	+22	+22	+2	+22	0
88	−4.9	+2.1	+8	+16	+18	0	+18	−1
89	−4.8	+2.1	+19	+17	+23	−11	+22	−12
90	−4.9	+1.7	+8	+16	+18	−2	+18	−2
91	−4.9	+1.7	+12	+17	+20	−6	+20	−6
92	−4.6	+1.6	0	+22	+21	+7	+21	+7
93	−4.8	+1.0	−2	+21	+20	+6	+21	+3
94	−4.6	+1.0	+15	+9	+12	−13	+9	−15
95	−5.2	−0.3	+4	+4	+3	−5	+4	−4
96	−5.4	−0.3	+3	+21	+21	−4	+20	−6
97	−5.1	−0.3	+13	+12	+11	−15	+10	−15
98	−5.1	−0.6	+2	+22	+21	−5	+21	−4
99	−5.2	−0.8	+5	+9	+8	−7	+8	−7
100	−5.9	−3.0	−18	+11	+18	+11	+20	+8
101	−5.6	−3.0	−20	+17	+25	+9	+26	+4
102	−4.9	−3.0	−25	+14	+25	+13	+27	+8
103	−5.6	−4.6	−15	+15	+21	+3	+21	−3
104	−5.6	−4.6	−9	+12	+15	0	+14	−4
105	−5.4	−4.6	−20	+9	+20	+9	+22	+3
106	−5.1	−4.6	−16	+11	+18	+6	+19	−1
107	−5.1	−4.5	−20	+8	+19	+9	+21	+3
108	−5.2	−4.5	−24	−4	+11	+21	+17	+17
109	−4.3	−4.0	−9	+12	+15	0	+15	−3
110	−4.0	−5.4	−13	+4	+13	+4	+13	−4
111	−4.5	−5.6	−15	+18	+23	−5	+18	−15
112	−4.5	−5.7	−27	+5	+24	+12	+27	0
113	−4.5	−6.0	−14	+16	+21	−5	+17	−14
114	−4.3	−6.2	−21	+17	+27	−3	+21	−17
115	−4.3	−6.4	−31	+7	+30	+12	+31	−7
116	−4.9	−6.2	−22	+2	+18	+13	+22	+1
117	−5.6	−8.1	−21	+2	+19	+10	+21	0
118	−5.4	−8.0	−22	+7	+22	+6	+23	−5
119	−5.6	−8.3	−18	−7	+11	+16	+18	+7
120	−5.4	−8.3	−14	−5	+6	+13	+11	+10
121	−3.2	−8.4	−32	−2	+29	+14	+32	0
122	−3.0	−8.4	−24	−7	+19	+16	+25	+3
123	−3.0	−8.4	−29	−14	+22	+23	+31	+9
124	−3.0	−8.6	−27	−13	+22	+20	+29	+7
125	−2.9	−8.6	−28	−2	+27	+7	+28	−2
126	−2.7	−8.4	−27	+1	+26	+5	+26	−7
127	−2.2	−8.4	−20	−3	+19	+8	+20	−4
128	−2.4	−8.3	−27	−15	+21	+22	+30	+8
129	−2.2	−8.1	−28	−1	+26	+9	+27	−8
130	−2.4	−8.1	−21	−5	+19	+11	+21	0
131	−1.9	−7.8	−22	−11	+20	+15	+24	+1
132	−1.6	−7.6	−27	−10	+25	+15	+29	0
133	−1.7	−7.5	−28	−3	+27	+8	+26	−11
134	−1.7	−7.5	−27	−6	+25	+11	+27	−8
135	−1.6	−7.3	−21	−11	+18	+15	+24	+1
136	−1.6	−7.5	−15	−3	+14	+7	+15	−3
137	−1.1	−6.8	−13	−12	+11	+14	+17	+5
138	−1.3	−6.7	−15	−8	+15	+8	+17	0
139	−2.7	−5.1	−27	−18	+16	+28	+29	+14
140	−2.5	−4.8	−22	−6	+17	+15	+23	0
141	−2.4	−4.5	−11	−5	+7	+9	+12	+2
142	−2.9	−4.6	−14	−2	+11	+9	+14	0

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
143	−3′.2	−4′.1	−0″.016	+0″.005	+0″.016	+0″.005	+0″.017	−0″.003
144	−3.3	−4.0	−10	0	+8	+6	+10	+2
145	−3.2	−3.8	−16	−3	+10	+12	+15	+6
146	−3.2	−3.5	−15	+9	+17	+3	+18	0
147	−3.2	−3.3	−13	+3	+11	+7	+13	+3
148	−3.0	−3.3	−12	+5	+13	+3	+13	−1
149	−4.5	−2.5	−12	+13	+17	+5	+18	+1
150	−4.5	−2.2	−22	+20	+28	+11	+29	+5
151	−4.1	−2.5	−17	+18	+24	+7	+24	+3
152	−4.0	−2.5	−15	+16	+21	+5	+22	+2
153	−3.5	−2.4	−20	+19	+27	+5	+27	+4
154	−3.8	−1.9	−18	+3	+15	+10	+15	+10
155	−3.7	−1.9	−2	+16	+15	−6	+15	−6
156	−2.5	−2.1	−8	+10	+13	−1	+13	−2
157	−2.4	−1.7	−17	+13	+21	+4	+21	+4
158	−2.9	−1.1	−4	+3	+4	+3	+5	+2
159	−3.3	−0.2	−1	+12	+12	−1	+12	−3
160	−3.5	−0.2	−5	+23	+23	+4	+23	0
161	−3.5	−0.2	−6	+13	+14	+6	+14	+3
162	−3.5	−0.2	−10	+16	+17	+9	+18	+6
163	−3.3	0.0	−8	+11	+12	+8	+13	+6
164	−3.3	0.0	+7	0	0	−7	−2	−7
165	−3.3	−0.2	+1	+15	+15	−1	+14	−4
166	−4.0	+2.1	−4	+26	+15	+21	+23	+12
167	−3.8	+2.2	+3	+8	+8	+1	+8	0
168	−3.7	+2.2	+6	+10	+12	0	+10	−6
169	−3.5	+7.6	+19	+14	+23	+5	+24	+1
170	−3.5	+7.8	+17	+18	+22	+11	+24	+7
171	−3.0	+11.1	+1	+10	+3	+10	+6	+8
172	−3.0	+11.1	+9	−2	+8	−5	+7	−6
173	−2.9	+11.1	+23	+5	+24	0	+23	−6
174	−2.7	+11.1	+19	−7	+17	−10	+15	−13
175	−2.9	+11.1	+19	−1	+18	−6	+16	−9
176	−2.9	+11.3	+23	−2	+21	−8	+20	−12
177	−0.6	+11.8	+30	−1	+30	−3	+29	−9
178	−0.6	+11.9	+16	+11	+16	+10	+18	+6
179	−0.2	+11.9	0	−1	0	−1	0	−1
180	−0.2	+11.8	+9	−1	+9	−2	+9	−4
181	−0.3	+11.6	+25	+4	+25	+3	+25	−3
182	−0.5	+11.4	+25	+1	+25	+1	+24	−5
183	−0.5	+11.3	+30	−1	+30	−3	+28	−9
184	+1.4	+9.2	+16	+22	+13	+24	+21	+17
185	+1.7	+9.1	+19	+3	+18	+7	+19	−1
186	+1.7	+9.1	+17	−2	+17	0	+15	−8
187	+1.4	+9.1	+26	+9	+25	+12	+27	+1
188	+1.3	+8.9	+22	+14	+20	+17	+25	+5
189	+1.1	+8.9	+11	+7	+10	+8	+13	+3
190	−0.6	+5.2	+18	+15	+20	+12	+23	−4
191	−0.6	+5.1	+24	+19	+26	+16	+30	−4
192	−0.6	+5.2	+16	+10	+17	+8	+18	−4
193	−1.3	+5.1	+18	+18	+23	+11	+25	−3
194	−1.3	+4.9	+9	+9	+12	+6	+13	0
195	−1.4	+4.9	+2	+15	+14	+7	+13	+8
196	−1.1	+4.8	+8	+18	+12	+16	+18	+7
197	−0.6	+4.3	+3	+10	+6	+9	+10	+3
198	−2.5	+3.5	+10	+21	+21	+11	+23	0
199	−2.7	+1.4	+6	+22	+23	+3	+22	−6
200	−2.2	+1.4	+4	−3	0	−5	−3	−4
201	−1.9	+1.4	+9	+14	+16	+1	+14	−8
202	−1.9	+0.2	+4	+13	+12	−5	+9	−10
203	−2.4	−0.5	−13	+12	+14	+10	+16	+8
204	−1.4	−0.8	−11	+12	+16	+3	+16	+1
205	−1.7	−1.1	−9	+10	+14	+1	+14	+1
206	−1.6	−2.9	−16	+8	+18	0	+17	−6
207	−1.1	−2.9	−12	0	+11	+4	+12	0
208	−1.0	−2.9	−18	−6	+15	+12	+18	+4
209	−1.3	−4.0	−22	−14	+18	+19	+23	+12
210	−1.1	−4.0	−19	−7	+16	+13	+19	+5
211	−1.1	−4.1	−23	−15	+19	+20	+24	+12
212	−1.4	−4.8	−14	−14	+9	+17	+17	+10
213	−1.3	−4.8	−16	−2	+15	+5	+16	−2
214	−0.5	−1.4	−5	+5	+6	−3	+7	−2

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
215	−0.′3	−1.′3	−0.″010	−0.″005	+0.″009	+0.″007	+0.″008	+0.″008
216	−0.2	−1.3	+2	−14	−4	+14	−8	+11
217	0.0	−1.4	−12	+5	+12	−7	+13	0
218	0.0	−1.4	−16	−9	+16	+10	+13	+13
219	+0.2	−1.7	−17	−16	+18	+14	+14	+18
220	+0.2	−2.9	−21	−10	+22	+8	+23	0
221	+0.2	−3.0	−10	−2	+10	+2	+10	−1
222	+0.3	−3.0	−11	−16	+12	+14	+17	+9
223	+0.2	−3.8	−8	−18	+9	+17	+15	+13
224	+1.1	−4.1	−27	−21	+32	+13	+34	−1
225	+1.1	−4.5	−28	−11	+30	+2	+27	−13
226	+1.0	−4.6	−23	−13	+25	+8	+26	−4
227	+0.6	−5.6	−22	−17	+25	+13	+27	−7
228	−0.2	−7.2	−26	−14	+26	+14	+29	−4
229	−0.2	−7.3	−29	−10	+28	+11	+29	−10
230	0.0	−7.2	−25	−4	+25	+4	+23	−11
231	+0.6	−7.0	−18	−10	+19	+8	+20	−4
232	+0.6	−6.8	−34	−23	+36	+19	+41	−5
233	+1.1	−6.4	−21	−10	+23	+4	+22	−7
234	+1.3	−6.0	−19	−8	+20	+3	+18	−9
235	+1.7	−5.9	−27	−31	+34	+23	+41	−1
236	+2.1	−5.7	−12	−26	+20	+21	+28	+7
237	+1.7	−5.9	−18	−18	+22	+12	+25	+1
238	+1.7	−6.0	−15	−12	+17	+8	+19	−3
239	+1.6	−6.5	−11	−21	+15	+18	+23	+7
240	+1.6	−6.7	−22	−19	+26	+13	+28	−7
241	+1.7	−6.7	−17	−14	+20	+10	+21	−5
242	+1.0	−6.8	−13	−11	+14	+9	+17	−2
243	+1.3	−7.2	−21	−9	+23	+3	+21	−10
244	+1.3	−7.2	−21	−16	+23	+12	+26	−2
245	+0.6	−7.6	−27	−24	+35	+11	+36	−2
246	+0.8	−7.5	−21	−12	+22	+9	+22	−8
247	+1.3	−7.5	−19	−18	+22	+14	+26	0
248	+1.4	−7.3	−8	−15	+12	+13	+16	+4
249	+1.6	−7.8	−27	−6	+28	0	+22	−17
250	+0.6	−12.4	−23	−2	+23	−1	+19	−13
251	+0.8	−12.2	−29	+1	+29	−5	+24	−17
252	+1.0	−12.2	−22	−7	+22	+4	+21	−9
253	+0.8	−12.4	−42	−16	+44	+11	+44	−11
254	+1.3	−12.2	−39	−7	+39	+3	+35	−18
255	+1.4	−12.2	−30	−1	+30	−3	+25	−17
256	+3.0	−4.0	−16	−19	+24	+6	+24	−7
257	+3.0	−3.8	−14	−13	+19	0	+17	−10
258	+2.9	−3.8	−12	−23	+24	+11	+25	+6
259	+2.1	−3.0	−19	−15	+24	+3	+23	−7
260	+2.2	−3.2	−7	−17	+15	+10	+18	+1
261	+2.5	−2.9	+1	−7	+4	+6	+6	+4
262	+2.9	−2.7	−1	−12	+9	+7	+12	0
263	+2.5	−1.7	−11	−10	+14	−4	+12	−9
264	+2.7	−1.7	−1	−25	+23	+11	+25	−1
265	+1.4	−1.3	−8	−11	+13	+2	+14	0
266	+1.1	−1.1	−24	−10	+24	−10	+26	0
267	+2.2	−0.2	+8	−10	+9	+9	+10	+8
268	+2.5	+0.2	+11	−10	+10	+11	+13	+6
269	+2.2	+0.2	+7	−19	+19	+6	+20	0
270	+2.5	+0.6	+16	−9	+12	+14	+17	+7
271	+2.5	+1.1	+5	−10	+11	+1	+10	−3
272	+2.5	+1.7	+14	+3	+5	+14	+10	+10
273	+2.4	+1.7	+15	+10	+1	+18	+8	+16
274	+2.7	+2.5	+25	+1	+15	+20	+23	+10
275	+2.7	+2.7	+12	−1	+10	+7	+12	+3
276	+2.5	+2.9	+17	−6	+17	+6	+18	−4
277	+2.2	+2.9	+14	+3	+9	+11	+13	+6
278	+2.4	+3.0	+16	+1	+12	+11	+16	+2
279	+2.5	+3.0	+8	+6	+2	+10	+7	+7
280	+1.6	+2.7	+14	+5	+9	+11	+14	+4
281	+1.3	+2.4	+1	+9	−3	+9	+4	+8
282	+1.3	+2.2	+20	+14	+11	+22	+24	+6
283	+0.2	+1.4	+7	+10	+6	+11	+12	0
284	+0.2	+1.6	+16	+21	+12	+24	+26	+2
285	0.0	+1.4	+16	+2	+15	+4	+14	−8
286	0.0	+1.7	+11	+6	+12	+5	+12	−5

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
287	−0.′2	+1.′9	+0.″018	+0.″022	+0.″020	+0.″020	+0.″029	+0.″001
288	+1.0	+5.9	+13	+5	+13	+6	+14	−2
289	+1.6	+6.0	+23	+20	+18	+25	+29	+8
290	+1.6	+6.2	+21	+12	+18	+16	+24	+5
291	+1.9	+6.2	+24	+13	+20	+19	+27	+4
292	+2.4	+6.8	+25	−1	+24	+6	+25	−2
293	+2.4	+6.8	+27	−2	+26	+8	+26	−6
294	+2.7	+7.0	+20	−1	+19	+5	+19	−5
295	+2.7	+7.0	+21	+5	+17	+13	+21	+2
296	+3.7	+7.8	+20	+4	+17	+12	+20	+4
297	+3.7	+8.0	+9	+5	+5	+9	+9	+5
298	+3.3	+12.4	+25	−13	+27	−6	+24	−14
299	+3.5	+12.2	+14	−11	+17	−7	+13	−13
300	+3.8	+12.2	+28	+1	+26	+10	+28	−1
301	+5.9	+12.1	+3	−31	+18	−26	+8	−30
302	+4.9	+9.1	+16	−9	+18	−1	+17	−7
303	+4.9	+8.9	+32	−19	+37	−2	+32	−19
304	+4.9	+8.7	+28	0	+24	+14	+28	+3
305	+4.9	+8.7	+9	−9	+12	−5	+9	−9
306	+4.6	+7.0	+20	−20	+27	−7	+24	−15
307	+4.5	+6.8	+20	−9	+22	+4	+22	−3
308	+4.6	+6.8	+29	−6	+27	+12	+30	−1
309	+4.3	+6.7	+18	−10	+20	+2	+20	−3
310	+4.3	+6.4	+21	−17	+27	−2	+24	−12
311	+4.9	+5.9	+13	−16	+20	−5	+17	−11
312	+5.1	+6.0	+11	−17	+19	−7	+17	−11
313	+5.2	+6.2	+15	−13	+20	0	+19	−7
314	+5.4	+6.2	+8	−9	+12	−1	+11	−5
315	+7.0	+6.4	+25	−11	+25	+11	+27	+3
316	+7.2	+6.2	+4	−11	+11	−4	+9	−7
317	+7.2	+6.4	+18	−19	+26	+2	+25	−7
318	+7.3	+6.4	+28	−25	+38	+4	+37	−9
319	+7.0	+5.9	+14	−22	+26	−4	+24	−10
320	+7.0	+5.9	+17	−17	+24	+3	+23	−5
321	+5.9	+5.1	+9	−16	+17	−4	+16	−9
322	+5.9	+4.9	+16	−18	+24	0	+23	−7
323	+6.2	+4.9	+17	−16	+23	+4	+23	−4
324	+6.8	+4.3	+17	−11	+19	+7	+20	0
325	+6.8	+4.1	+13	−16	+20	+2	+20	−3
326	+6.8	+4.1	+14	−15	+20	+5	+21	0
327	+5.2	+2.7	+24	−15	+24	+15	+28	+5
328	+5.4	+2.7	+24	−19	+29	+11	+30	−1
329	+5.2	+2.5	+19	−16	+23	+9	+25	0
330	+4.3	+0.8	+11	−6	+7	+9	+9	+7
331	+4.5	+0.8	+8	−4	+6	+7	+7	+6
332	+4.8	+0.5	+11	−14	+15	+10	+17	+7
333	+3.3	−0.2	+9	−18	+17	+10	+19	+6
334	+3.3	−0.3	+17	−13	+10	+19	+16	+15
335	+4.0	−0.3	+8	−8	+7	+8	+9	+7
336	+4.1	−0.3	+11	−10	+8	+12	+11	+9
337	+4.1	−0.5	+28	−1	−2	+28	+6	+27
338	+4.0	−1.4	−16	−9	+13	−13	+9	−16
339	+5.1	−1.3	+7	−17	+15	+11	+18	+6
340	+4.9	−1.4	+4	−18	+15	+10	+18	+4
341	+4.9	−1.6	−2	−20	+20	+3	+20	−3
342	+5.1	−1.6	+1	−13	+12	+6	+13	0
343	+4.9	−2.1	−22	−18	+25	−13	+18	−22
344	+5.1	−2.1	−1	−20	+19	+6	+20	−2
345	+5.1	−2.1	−4	−16	+16	+2	+16	−3
346	+4.8	−3.2	−10	−22	+24	+4	+23	−9
347	+4.6	−3.3	−9	−17	+18	+3	+18	−8
348	+4.6	−3.5	−12	−18	+22	−1	+19	−11
349	+4.8	−4.9	−17	−17	+24	−1	+21	−12
350	+4.8	−5.1	−18	−24	+30	+4	+28	−11
351	+4.6	−5.4	−20	−11	+22	−8	+18	−14
352	+5.9	−5.2	−12	−20	+23	+4	+22	−7
353	+5.4	−6.7	−15	−24	+26	+11	+28	−3
354	+5.7	−6.7	−27	−18	+32	−4	+28	−16
355	+5.6	−6.8	−26	−22	+34	0	+30	−16
356	+5.7	−7.8	−25	−20	+32	+1	+30	−12
357	+5.7	−7.8	−28	−14	+31	−6	+25	−19
358	+6.7	−6.5	−6	−23	+21	+11	+24	0

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
359	+9′.9	−5′.2	−0″.029	−0″.024	+0″.035	−0″.015	+0″.031	−0″.022
360	+8.7	−4.8	−16	−29	+33	0	+31	−11
361	+9.2	−4.8	−17	−18	+24	−8	+20	−14
362	+9.2	−4.3	−22	−28	+35	−7	+31	−17
363	+6.2	−3.5	−2	−17	+16	+5	+17	−3
364	+5.6	−3.2	−3	−28	+26	+10	+28	−4
365	+5.9	−2.9	−5	−20	+20	+5	+20	−5
366	+6.2	−0.2	+5	−16	+15	+7	+16	+4
367	+6.4	−0.2	+95	−98	....	....	....	....
368	+6.4	0.0	+2	−21	+21	+2	+21	−2
369	+5.4	+0.2	+2	−18	+18	+2	+18	−1
370	+5.4	+0.3	+5	−16	+16	+5	+17	0
371	+10.8	−2.5	−11	−28	+29	−6	+29	−10
372	+11.1	−2.5	−6	−29	+29	0	+29	−3
373	+11.4	−2.2	−13	−23	+24	−10	+24	−11
374	+11.3	−2.1	−11	−21	+22	−8	+22	−9
375	+9.4	−0.2	+10	−17	+16	+11	+18	+9
376	+9.4	+0.2	+8	−13	+13	+8	+14	+5
377	+9.1	+0.3	+20	−21	+22	+20	+24	+17
378	+9.1	+0.5	−1	−24	+24	−3	+23	−5
379	+10.2	+1.1	+3	−22	+22	+2	+22	+2
380	+10.2	+1.4	+5	−19	+20	+2	+20	+2
381	+10.5	+1.1	+2	−12	+12	0	+12	+1
382	+10.7	+2.1	+3	−14	+14	+1	+14	0
383	+10.7	+2.4	+5	−22	+23	+1	+23	0
384	+10.5	+2.4	−5	−16	+14	−9	+14	−9
385	+10.8	+4.0	+1	−25	+24	−7	+23	−9
386	+11.0	+4.1	+2	−13	+13	−3	+13	−4
387	+9.9	+5.6	+13	−19	+23	+1	+23	−2
388	+9.9	+5.6	+26	−17	+28	+14	+30	+8
389	+10.0	+5.7	+16	−19	+24	+6	+25	−1
390	+10.0	+5.9	+11	−34	+35	−8	+32	−15
391	+10.5	+6.2	+7	−21	+22	−3	+21	−7
392	+10.7	+6.2	+6	−24	+24	−7	+23	−10
393	+10.8	+6.0	+6	−35	+33	−12	+31	−17
394	+11.0	+5.7	+16	−19	+24	+6	+25	+1
395	+11.0	+6.2	+20	−20	+27	+8	+28	+1
396	+11.4	+6.2	+6	−35	+34	−11	+32	−16
397	+11.8	+6.5	+21	−18	+26	+10	+27	+4
398	+11.9	+6.2	0	−19	+17	−8	+16	−10
399	+8.9	+6.7	+8	−26	+26	−9	+20	−18
400	+6.5	+9.2	+0.017	−0.009	+0.019	+0.001	+0.019	−0.004

a) The mean rotational component is $+0''.020 \pm 0''.001$; the mean radial component, $+0''.003 \pm 0''.001$. There seems, however, to be a considerable increase of motion with distance from the center. We have

$r = <3'$	$\mu_{\text{rot.}} = +0''.012$	$\mu_{\text{rad.}} = +0''.006$	$n = 26$
$3' - 6'$	$+0.015$	$+0.006$	100
$6 - 9$	$+0.021$	$+0.005$	103
$9 - 12$	$+0.022$	$+0.002$	98
$12 - 15$	$+0.024$	-0.004	69
>15	$+0.024$	-0.015	3

These rotational components correspond to hypothetical periods of from 60,000 to 240,000 years.

b) The mean stream component is $+0\overset{''}{.}020 \pm 0\overset{''}{.}001$, with a transverse component $-0\overset{''}{.}003 \pm 0\overset{''}{.}001$. With the increasing distances from the center we find

$r=$	$\mu_{\text{stream}}=$	$\mu_{\text{trans.}}=$	$n=$
$<3'$	$+0\overset{''}{.}013$	$+0\overset{''}{.}002$	26
$3'-6'$	$+0.017$	$+0.001$	100
$6-9$	$+0.022$	-0.003	103
$9-12$	$+0.023$	-0.004	98
$12-15$	$+0.023$	-0.009	69
>15	$+0.021$	-0.019	3

With these measures on M 33 we have reached the end of the work which can at present be done advantageously on the internal motions in spirals. Only two or three other spirals were photographed with the 60-inch reflector between 1910 and 1912, and these objects do not show images sharp enough to promise any further advance in our knowledge of internal motions. Since it seems wise to defer further measures until plates taken with the 100-inch Hooker telescope become available, it is appropriate to summarize and discuss the results thus far obtained.

Seven spirals have been measured, viz., M 33, 51, 63, 81, 94, 101, and N.G.C. 2403, all of which definitely show an internal motion, best interpreted as a motion outward along the arms. The question as to the reality of the displacements, which are very small, may be answered first.

a) The instrument with which the photographs were taken was for most of the plates the 25-foot arrangement of the 60-inch reflector. That the displacements are instrumental is very improbable; there is no reason why the old plates should differ in appearance from the new ones in such a way as to produce a displacement of the nebular points with respect to the comparison stars corresponding to rotational or stream motion. It is true that in practically all cases the comparison stars were in the mean brighter than the nebular points. This might give rise to a magnitude error, but such an error could produce only a bodily shift of the nebular points with respect to the comparison stars, or a radial shift due to curvature of the field and the smaller mean distance of the nebular points from the center than of the comparison stars.

But relative motion along the arms of the spirals cannot conceivably be caused by the instrument. Moreover, it would be strange if the instrument produced a left-handed twist for the left-handed spirals, M 51, 81, and 101, and a right-handed twist for the right-handed spirals, M 33, 63, 94, and N.G.C. 2403. Further, three of the plates measured were taken with the 36-inch reflector of the Lick Observatory, and two with the 80-foot arrangement of the 60-inch reflector. Last of all, similar motions have been found in M 51 by Kostinsky, Lampland, and Schouten, from observations with three other smaller instruments.

b) The possibility that the displacements are due to a difference in quality in old and new plates is also extremely small. A difference in quality of the images or in the density of the plates might cause a slight difference in the bisections, because of asymmetry in the images of the nebular points. As a whole, the older plates taken by Ritchey are of extremely good quality, and many of the newer plates are equally good. Of the Lick photographs of M 101, the 1908 plate was considerably better than that of 1899. Of the two plates of M 33, taken at the 80-foot focus of the 60-inch reflector, the later plate is also of considerably better quality. As far as the density of the photographs is concerned, the exposure times of the new plates taken with the 60-inch reflector have all been made so nearly equal to those of the old plates, that no difference in the density can be seen; neither the quality nor the density can therefore account for the displacements found.

c) In measuring the plates, three machines were used: the Zeiss stereocomparator, the new stereocomparator built in our instrument shop, and an auxiliary instrument used for some test measures. For the last, two plates of M 101 were mounted side by side on the moving stage of an ordinary measuring machine, which was fitted with an extra microscope, one for each plate, thus permitting differential measures. It is clear that defects in the optical system of the stereocomparator could never reveal themselves as a rotatory motion of the nebular points, without equally affecting the comparison stars. Moreover, the effect of such defects would be eliminated in the reductions. Further in the case of M 33, 81, 101, and N.G.C. 2403, three or more plates were measured. As an

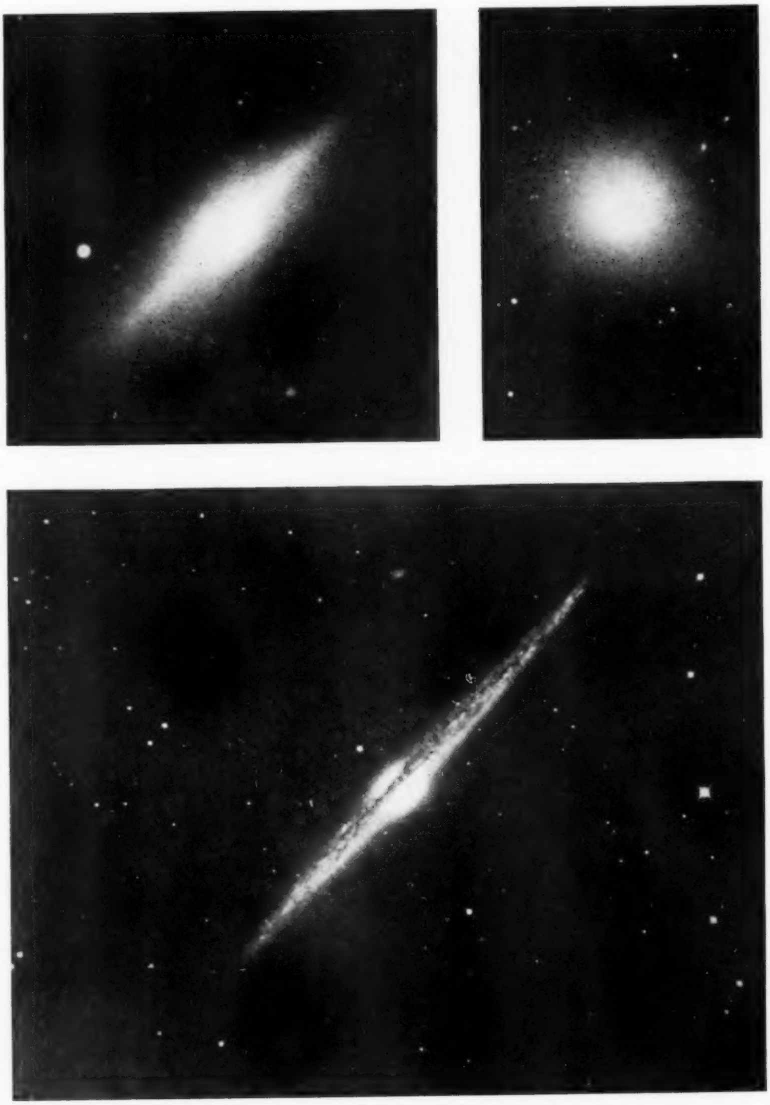
extra precaution, care was taken in all these cases to measure the plates partly with the older plate in the left-hand, and partly in the right-hand plate-carrier of the stereocomparator. This precaution would also eliminate any influence due to a possible curvature in the rails, either horizontal or vertical, that guide the plate-carriers. Finally, several other plates measured for proper motion in both stereocomparators have never shown any rotational effects whatsoever.

d) The measures were practically all made by the author. Mr. Nicholson, however, was kind enough to make enough check measures on M 101, both with the Zeiss stereocomparator and with the auxiliary machine, to avoid any doubt as to the results. Lately Mr. Lundmark has carried out measures on M 33, and his results seem, as a whole, to corroborate those by the author. Finally, there are the measures by Kostinsky, Lampland, and Schouten mentioned under (*a*).

If, then, the results obtained in all seven spirals measured are to be taken as real displacements of the nebular points with respect to the comparison stars, as apparently must be done, there are only two possible explanations: either the comparison stars show a vortex-motion around the spirals, which to say the least is very improbable, or the spirals show a rotatory or stream motion. The beautiful work of Jeans has given us reason even to anticipate such motions. He has shown that, in a highly compressible mass of rotating gas, as a gaseous nebula undoubtedly is, we must expect shrinking as a result of radiation. The rotation must become faster and faster, until the nebula takes on the shape of an oblate spheroid; the spheroidal shape, however, is soon departed from, and at a certain critical speed a sharp equatorial edge is developed. With still further contraction and accordingly higher speed of rotation, the particles which form the sharp edge are left behind. It does not matter if they are thrown off at all points equally or not. Neither rings of gas nor jets of gas, which must appear at two antipodal points when other heavenly bodies are present, can be stable; they must tend to form condensations or nuclei. In the sky we find plenty of examples of development such as just described. Three stages are shown in Plate XX, N.G.C. 4486, 3115, and 4565, examples

PLATE XX

a *b* *c*



a) N.G.C. 4486; *b*) N.G.C. 3115; *c*) N.G.C. 4565

of the nearly spherical nebula,[1] the lenticular figure, and the spiral seen edgewise. That N.G.C. 4565 is really a spiral cannot be doubted. The universe has revealed so many spiral nebulae inclined at different angles to the tangential plane of the celestial sphere that it is quite certain M 33 and N.G.C. 4565 belong to the same class of objects. That the spirals show rotation around the smaller axis has been proved spectroscopically in six or seven spirals whose planes are inclined to the celestial sphere. No object tested thus far has shown a lack of rotation. It thus seems only logical to interpret the displacements observed in those spirals which are viewed perpendicularly to their planes as motions corresponding to the rotation found spectroscopically. In only two of the spirals measured has it been possible to detect spectroscopic motion as well. Some of the others form so nearly an angle of 90° with the line of sight that no spectroscopic results can be expected, while others are so faint that no spectroscopic results have as yet been secured. For M 81 Wolf[2] has observed rotation in the central nebulous part within which it has been impossible to secure measures from the photographs. For M 33, however, it has been possible to observe both the radial velocity and the motion perpendicular to the line of sight for the same point, viz., the bright knot 10′ *nf* the nucleus. Taking into account the probable inclination of the nebula with respect to the tangential plane of the celestial sphere, we can gain some idea of the order of the parallax of the nebula; the result is $\pi = 0\overset{''}{.}0005$.

Happily there are other means for obtaining an idea of the distances of spiral nebulae. Jeans has shown that mathematical theory not only predicts that condensations will form in the arms of the nebula, but also predicts how far apart these condensations will be. The comparison of these calculated mean distances with the mean distances as they appear in the sky provide a second means of estimating the distances of the nebulae. In this way Jeans[3] estimated

[1] Although it cannot be decided whether this particular nebula is a spherical object or a disk perpendicular to the line of sight, the large percentage of round or nearly round nebulae found among the structureless objects makes it probable that many of these are spherical.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 162, 1914.

[3] *The Nebular Hypothesis and Modern Cosmogony: Being the Halley Lecture delivered on 23d May 1922*, Oxford, 1923.

the parallax of the Andromeda nebula to be 0.″0006; of M 101, 0.″0011; and of M 51, 0.″0065.

We have further possibilities of estimating the mean parallax of the larger spiral nebulae. Curtis[1] gives 0.″033 as the average annual motion of sixty-six large spiral nebulae. In a recent paper[2] I have shown that Curtis' results cannot be due to the influence of accidental errors as was once assumed by Curtis himself. Comparing this mean proper motion with the mean radial velocity of about thirty objects (600 km/sec.) for which the line-of-sight motion has been determined, we derive a mean parallax of 0.″00026. Finally, we may quote again the values recently derived for the mean parallax of sixty-seven and eighty-two spirals whose motions were determined by Curtis and Lundmark, respectively. Using 600 km/sec. for Campbell's K-term, the mean parallaxes are 0.″00013 and 0.″00015, respectively.

All this material seems to point to parallaxes for the larger spiral nebulae between a few ten-thousandths and a few thousandths of a second of arc. With such values the diameters of the spirals range from a few light-years to several hundred light-years. Since our present estimates of the Milky Way system vary, according to different authorities, from 20,000 to 300,000 light-years, it is clear that the present material indicates that the spirals, while enormous in size as compared with our solar system, are not at all comparable with the Milky Way system.

In concluding, I wish to express my sincere thanks to Messrs. Ritchey, Pease, Humason, and Duncan, who have secured the necessary photographs to carry out this work, and to Miss Davis and Mrs. Marsh, of the Computing Division, who have assisted in the numerous reductions involved.

MOUNT WILSON OBSERVATORY
March 1923

[1] *Publications of the Astronomical Society of the Pacific*, **27**, 217, 1915.

[2] *Mt. Wilson Contr.*, No. 243; *Astrophysical Journal*, **56**, 208, 1922.

THE VACUUM-SPARK SPECTRUM OF SILICON

By R. A. SAWYER and R. F. PATON

ABSTRACT

Vacuum-spark spectrum of silicon, λ 6700–λ 2100.—Silicon electrodes less than a millimeter apart were mounted in a suitably designed brass box with a quartz window, the best obtainable vacuum was produced, and a highly condensed spark was obtained by the use of 70,000 volts. Lines of iron, aluminum, calcium, and oxygen and the strongest lines of hydrogen, nitrogen, copper, zinc, and titanium appeared as impurities. The wave-lengths of 227 lines attributed to silicon in the region λ 6700–λ 2100, together with the measurements of other observers, are given in Table I. An additional 75 lines, listed in Table II, are given as doubtful, since they were faint lines appearing on only one plate. In the intervals λ 6700–λ 5500 and λ 4070–λ 3400 the wave-lengths are believed to be accurate to within 0.2 A and in the intervals λ 5500–λ 4070 and λ 3400–λ 2100 to within 0.1 A.

I. INTRODUCTION

Silicon is one of the most widely distributed of all the chemical elements, but is difficult to isolate in a pure state. Moreover the slight conductivity of pure silicon renders its use with ordinary light sources very difficult. It is not surprising, then, that most of the earliest spectroscopic work on silicon was done with its compounds only. Kayser[1] has summarized all of the important work on silicon up to 1912. The first work Kayser mentions dates back as far as 1859, when Plücker investigated the light given off by an electrical discharge in various gases and vapors in a Geissler tube. Using silicon chloride, Plücker found four lines which he attributed to silicon. Of those who followed, Rowland was the first to point out the presence of silicon in the sun, and Lockyer identified silicon lines in the spectra of many of the stars. Up to 1912, however, none of the work on silicon seems to have been particularly satisfactory.

The most recent work on the spectrum of silicon was done by Sir William Crookes[2] in 1914. He succeeded in obtaining some rather pure samples of silicon, and using a condensed spark between

[1] *Handbuch der Spectroscopie*, **6**, 478–494.

[2] *Proceedings Royal Society*, Series A, **90**, 512–520, 1914.

electrodes of this silicon, photographed the spectrum with a prism spectrograph. The rapid oxidation of the silicon made it very difficult to maintain the spark and necessitated long exposures. His work covered the region λ 6500 to λ 2100 and he published the wave-lengths of 43 lines, including most of the lines previously recorded. This investigation was the most thorough work that had been done on silicon up to that time. Since the work of Crookes, Fowler[1] has mentioned observing four lines in the visible region of the spectrum, not previously observed; and McLennan[2] has extended the investigation into the extreme ultra-violet, cataloguing some seventeen additional lines.

A comparison of the results of all the work that has been done on silicon brings out the fact that no two observers have the same list of lines, and also shows that even in the lines that have been recorded by several observers, the disagreement in wave-length seems almost unaccountably large. This disagreement may be partly accounted for, however, by the fact that many of the silicon lines are especially broad and hazy when produced at atmospheric pressure.

The importance of having accurate and complete results on the spectrum of silicon can well be realized when one considers the wide terrestrial distribution of this element, as well as the fact that its presence in many of the stars has been definitely established. Complete and accurate data on the spectrum of silicon are also desirable to throw new light on its spectral series, the details of which have not yet been completely worked out. With the discovery of the vacuum-spark, a new source of light became available that seemed to have particular advantages in this problem. The use of the vacuum-spark in extending the spectrum into the extreme ultra-violet,[3] had given results which led to the expectation that it might give new information in the rest of the spectrum. With this in mind, it was decided to photograph the vacuum-spark of silicon in the region covered by Crookes, hoping that the results might give more accurate information concerning the wave-lengths of some

[1] *Monthly Notices*, **74**, 196–197, 1916.

[2] *Philosophical Magazine*, **30**, 482–484, 1915.

[3] *Astrophysical Journal*, **52**, 286–300, 1920.

of the lines and even add to the already known list. In adapting the vacuum-spark to silicon, it was found necessary to use high voltage and extremely good vacua; but once obtained, the spark was brilliant, of a reddish yellow color, and resembled the vacuum-spark of carbon.

II. DESCRIPTION OF APPARATUS AND MANIPULATIONS

A 100,000-volt closed-core transformer gave the necessary voltage. Energy was supplied to the primary from a 110-volt power circuit, and controlled by a heavy rheostat in series with the primary. To prevent overheating the transformer and the silicon electrodes, a commutator switch was placed in the primary and adjusted so that the current was automatically turned on forty times a minute for about one half a second at a time. One terminal of the high voltage transformer was connected directly to one of the electrodes; the other terminal was grounded. By grounding the second electrode, the circuit was completed with only one line charged to a dangerous voltage.

In order to have a large amount of energy available when the spark passed, a capacity consisting of five glass condensers, immersed in oil and connected in series, was placed in parallel with the gap. This arrangement gave a capacity of the order of magnitude of one one-hundredth of a microfarad, while the voltage over any one condenser was not sufficient to puncture the glass dielectric.

The vacuum was obtained with two mercury diffusion pumps designed to work in series, and a small oil pump as a pre-pump. The vapor pressure of the mercury at room temperature was too great to permit a sufficiently high vacuum to be obtained. To reduce this vapor pressure, a glass trap was sealed in between the spark box and the pumps. When this trap was immersed in liquid air, the pressure due to any vapors present was sufficiently reduced so that vacua could be obtained that would remain non-conducting for sparking voltages of roughly 70,000 volts with a spark-gap length of less than one millimeter. It was found that, with the pumps working well, such a vacuum could be obtained in about five minutes. The pressure was then of the order of magnitude of one ten-thousandth of a millimeter of mercury or less.

Since the spark box was heated somewhat by the spark radiation, and the electrodes themselves heated to a white heat the instant the spark passed, frequent adjustments of the spark were necessary. To accomplish these adjustments without opening the box, the electrodes were mounted eccentrically in insulating plugs ground into brass tubes soldered to opposite ends of the box. The electrodes could then be rotated inside the box without affecting the vacuum. One electrode was mounted on a nut which ran on a shaft that could be rotated from the outside in such a way as to move the electrode in or out as desired, and thus regulate the distance between the electrodes. The box was made with one removable side. Leakage was prevented by the use of a rubber gasket and a rubber stopcock grease prepared for the purpose. This removable side made it possible to renew the electrodes and clean the windows through which the spark was photographed.

The electrodes themselves consisted of small pieces of elementary silicon mounted in brass clamps. Pure silicon is very hard and brittle, and the electrodes could be shaped to rough points only by chipping. The silicon was very kindly furnished by the Carborundum Company of Niagara Falls, who also furnished that used by Crookes and McLennan.

With the electrodes adjusted so that they were all but touching, a spark could be obtained with a comparatively poor vacuum. This spark was very feeble; for most of the energy available was dissipated by conduction through the gas remaining in the box. With the best vacuum that could be obtained, the distance between the electrodes could be made somewhat larger, but never more than about a millimeter for silicon electrodes. The spark under these conditions was extremely brilliant. The electrodes became white-hot almost instantly, and some of the silicon was vaporized, eventually fogging the windows through which the spark was photographed. If the power was kept on for more than a second at a time, occluded gases from the electrodes and their insulators were given off in sufficient quantity to stop the spark. In spite of the very refractory nature of silicon, the electrodes were worn away rapidly, and frequent adjustments and renewals were necessary.

To photograph the spark, two spectrographs were used, both of which were designed and built during the course of this research.

The region λ 6700 to λ 4070 was photographed by a glass spectrograph of the Littrow type. A 3-inch telescope objective lens with a focal length of roughly 24 inches was used. It was a cemented doublet and gave a reasonably flat field. The refracting angle of the prism was 60°, and its face about an inch and a half across. The entire region λ 6700 to λ 4070 was photographed on a 5-inch plate. Exposures with this instrument extended sometimes over 15-hour periods, making it necessary to house the entire spectrograph to prevent shifts due to changes in temperature.

The region λ 4100 to λ 2100 was photographed by a quartz spectrograph of a type first described by Fabry and Buisson.[1] In this instrument, the slit was mounted directly over the middle of the prism. A platinized concave mirror of 32-inch focal length served to collimate the light from the slit. The prism consisted of a cemented doublet of right and left quartz—the whole having an oblong face 1.75 inches high and a refracting angle of 60°. The light, after passing through the prism, was focused on the plate by a quartz lens whose focal length was 35.7 inches for the "D" lines. The use of only one quartz lens has the advantage of making the focal plane of the instrument more nearly perpendicular to the axis of the lens, since the collimating mirror insures parallelism of the beams of every wave-length incident on the prism. The inclination of the plate is due wholly to the variation with wave-length of the focal length of the camera lens. Thus, in this instrument, the photographic plate is at an angle of 45° to the axis rather than at 30°, as in an instrument of similar power but using a single quartz collimating lens. A shorter plate is thus used and yet the increased sharpness of the lines due to the lesser inclination more than offsets the shortness of the spectrum. With the quartz instrument, the whole region from λ 6000 to λ 2000 was photographed on a 10-inch plate. Only the region λ 4100 to λ 2000 was sufficiently dispersed, however, for satisfactory measurements. The copper line at λ 1944 has been photographed with this instrument, but no lines below λ 2100 were observed in the silicon spectrum.

For the region λ 6700 to λ 4070, radiation from the spark was focused on the slit with a lens. In order to be sure that the radia-

[1] *Journal de Physique*, **9**, 929, 1910.

tion from the iron comparison arc would enter the slit at exactly the same angle as that from the vacuum-spark, the arc was placed beyond the spark box and on the extension of the line joining the slit and the silicon spark gap. A short focus lens formed an image of the arc between the spark electrodes. This insured identity of source for the known and unknown spectrum. The iron arc was used as a standard, as very complete data on the wave-lengths of these spectral lines are now available. These data were taken from results obtained at Mount Wilson[1] and at Bonn,[2] Germany. For the quartz instrument, no external source comparison was necessary as it was found that the iron occurred as an impurity in the silicon. An iron comparison spectrum was photographed on the plate, however, to help in identifying the spark lines.

For the visible region, a special type of panchromatic plate, manufactured by Ilford, London, England, was found most satisfactory. Ordinary Seed plates were used in the ultra-violet.

In order to guard against any possible error due to changes in the optical set-up during any one exposure, the iron comparison arc radiation was admitted to the spectrograph at the beginning, at the end, and eight to ten times at intervals during the exposure. This precaution was not necessary, of course, with the ultra-violet plates, as sufficient standard lines existed as impurities in the silicon spark itself.

III. MEASUREMENTS AND RESULTS

The plates were measured on a large engine kindly made available by the Detroit Observatory of the University of Michigan. The reductions were made by using the Hartmann interpolation formula, and the final wave-lengths expressed in International Angstroms. The results show that the probable error for the region λ 6700 to λ 5500 is about 0.2 angstroms; from λ 5500 to λ 4070 less than 0.1 angstroms; from λ 4070 to λ 3400 about 0.2 angstroms; and from λ 3400 to λ 2100 less than 0.1 angstroms. It is unfortunate that only two plates were obtained with the quartz instrument and the

[1] *Astrophysical Journal*, **53**, 260, 1921.

[2] *Zeitschrift für Wissenschaftliche Photographie*, **12**, 207–235, 1913; and **19**, 149–157, 1919.

possible error is therefore larger in the region λ 4070 to λ 2100. The results from λ 6700 to λ 4070 are the mean observations from eight plates and are correspondingly more reliable.

In general, conclusions as to the accuracy of the present work were drawn from the variation in wave-lengths of the lines between individual plates, and from the accuracy with which determinations were made on impurity lines of known wave-length. It is of interest to note, in considering the accuracy, that interferometric measurements of wave-length have been made by Fabry and Buisson on three ultra-violet silicon lines.[1] The following table shows the comparison of their results with the present work.

Fabry and Buisson	Vacuum-spark	Intensity	Difference
2528.516	2528.50	5	0.016
2506.904	2506.90	4	0.004
2435.159	2435.17	5	0.011

Such an agreement as this leads to the belief that the error estimates given above are, at least for the strong lines, conservative.

Altogether over 700 lines have been measured in the spark spectrum of silicon, and of this number some 400 have been identified as due to the elements which occurred as impurities in the source. This leaves a list of more than 300 lines that have been ascribed to silicon. Of this number, 75 are classified as "doubtful" lines (see Table II). These lines are mostly very faint and have been observed on one plate only. Although the possible error in their wave-lengths may be somewhat larger than for the rest, and in many cases their actual existence may be in doubt, still they have been listed, and evidence is in favor of retaining them. The photographs were taken under varying conditions as to exposure time, voltage used on the spark, length of spark gap, plate developing, and kind of plate; and many other lines, equally faint, have been measured and found to be due to impurities. The main list (Table I) contains only those lines concerning which there can be little doubt. All possible impurities have been very carefully checked, and where there was any reasonable doubt, the line in question has been eliminated. There still remain in this list some 224 lines where previously less than 80 had been observed.

[1] *Astrophysical Journal*, **27**, 169, 1908.

The wave-lengths are given in International Angstrom Units, whereas those by other observers are in the Rowland System. In general, the wave-lengths agree fairly well. In some cases, however, there exist variations of more than an angstrom. Such disagreement is hard to explain, and no such disagreement has been found in the results from the various plates measured during the course of the present work. The lines in most cases are sharp. General fogging of the plate due to band spectra of nitrogen or carbon, such as occurs in the ordinary spark in air or in the carbon arc, is entirely absent. Faint lines that under ordinary conditions would be obliterated, are, under the conditions of the vacuum-spark, measurable.

Of the impurities that appeared, only a very few can be attributed to remnants of gases in the vacuum chamber. The hydrogen line, $H\alpha$, showed faintly on only a few plates; $H\beta$, on two plates; $H\gamma$, and the remaining hydrogen lines, not at all. A few of the strongest mercury lines were recorded, and five of the strongest nitrogen lines. The strongest copper and zinc lines appear, coming probably from the brass clamps in which the silicon was mounted—there was usually a little sparking from the clamps. The lines which appeared due to impurities in the silicon itself are more numerous. In the visible, a few of the strongest iron lines appeared, but in the ultra-violet practically the entire spark spectrum of iron was observed. The appearance of the iron spectrum made unnecessary the use of a separate iron arc comparison, and eliminated any possibility of lack of identity in position of the silicon and the comparison source. Titanium, which Crookes mentioned as the principal impurity, appeared not at all in the visible, while only a few of the very strongest lines showed faintly in the ultra-violet. The entire spectrum of aluminum appeared in the visible, together with all but a few of the fainter arc lines in the ultra-violet. Of the two new aluminum lines observed by Shallenberger,[1] the one at λ 4150 came out strongly on all the plates, but the other one, at λ 2907, showed not at all. All of the strong oxygen lines appeared faintly, denoting probably a trace of oxide in the sample. One boron line showed, and a few carbon lines. The sodium D

[1] *Physical Review*, **19**, 398–399, 1922.

lines appeared extremely faint. H and K of calcium appeared strong, together with the calcium line at λ 4227. Most of the other strong calcium lines appeared on some of the plates. The resonance lines of magnesium, barium, and strontium, described by De Gramont[1] as "raies ultimes," were also photographed. The character of the impurity lines showed that while arc lines certainly appear in the vacuum-spark, the spectrum which it gives tends to emphasize the spark lines.

There is, of course, the possibility that some of the new lines found are due to impurities and not to silicon. To guard against this possibility, the vacuum-spark spectra of aluminum and of carbon were photographed and checked against the silicon plates. Only the aluminum line, λ 4150, however, was thus eliminated. The vacuum-spark spectra of iron, calcium, magnesium, and titanium were photographed through most of this region by Miss Carter,[2] who did not report any new lines. As the principal impurities occurring in the source used in this work are thus covered, the possibility that any of the new lines are due to impurities seems slight.

In comparing the results obtained, with those of previous observers, the number of new lines found is surprising. With but one or two exceptions, all the lines that have ever been found, either in the arc or spark of silicon, have been photographed. A few of the lines ascribed by some to silicon have been assigned to impurities. Special note is made of these lines in Table I. The intensities are estimated arbitrarily, but give some insight into the nature of the lines, and agree in the main with others. Strangely enough, though, the most intense line in the ultra-violet, λ 2541.91, has been missed by some observers, and no one before found it to be the strongest.

Time has not permitted a study of the lines for possible series, but the spectrum certainly contains many doublets and some triplets of constant frequency difference. The most conspicuous is the set of doublets having a frequency difference of 60 waves per centimeter. Sixteen of these doublets have been observed. Among the triplets, the difference −74−34− occurs five times with

[1] *Comptes Rendus*, **171**, 1105–1109, 1920.

[2] *Astrophysical Journal*, **55**, 162, 1922.

TABLE I

Vacuum-Spark Spectrum of Silicon

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
2122.33	0					
2123.77	1			2123.0	2124.17	2124.163
2135.90	2					
2148.81	1					
2179.37	2					
2189.62	2					
2192.22	3					
2208.01	2	2208.05	2208.06	2208.8	2208.1	2208.048
2210.92	3	2210.94	2210.97	2211.0	2210.9	2210.987
2211.76	2	2211.76	2211.84	2212.0	2211.8	2211.839
2216.71	3	2216.76	2216.75	2217.2	2216.76	2216.882
2218.11	3	2218.15	2218.13	2218.7	2218.15	2218.227
			2218.97	2219.5	2219.5	
2228.86	2					
2287.06	3					
2299.86	2					
(1)	1		2303.15	2303.8	2303.3	
2308.21	1					
2325.35	2					
2334.48	1					
2338.97	2					
2346.80	2					
2349.54	1					
2350.26	0					
2353.19	1					
2356.35	3				2356.9	
2357.20	2					
2358.02	2					
2363.82	2					
2371.02	0					
2416.54	1					
2419.80	1					
2432.23	2					
2435.17	5	2435.25	2435.27	2435.6	2435.25	2435.212
(2)	2	2438.86	2438.87	2439.4	2438.86	2438.911
2443.47	1	2443.46	2443.47	2443.5	2443.46	2443.484
(2)	1		2443.91			
(2)	1		2446.63		2446.0	
2449.70	2					
2452.12	1	2452.22	2452.23	2452.5	2452.22	2452.219
2480.04	1		2478.41		2479.8	
2481.10	0					
2483.29	1					
2486.28	2					
2495.79	1					
2500.90	1					
2503.64	1					
2506.90	4	2506.99	2507.01	2506.8	2506.99	2507.055
2514.34	4	2514.42	2514.41	2514.3	2514.42	2514.406
2516.08	5	2516.21	2516.26	2516.0	2516.21	2516.131
2517.48	3					

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
2519.22	3	2519.30	2519.30	2519.2	2519.30	2519.276
2524.11	4	2524.21	2524.21	2524.3	2524.21	2524.110
2528.50	5	2528.60	2528.60	2529.0	2528.60	2528.585
2532.41	1	......	2532.45	2533.0	2533.2	......
(2)	1	......	......	2535.0	2534.7	......
2541.91	10	......	2541.90	......	2541.89	2541.970
2559.20	4	......	......	......	......	......
......	......	......	2568.8	2569.0	2568.8	......
2570.78	1	......	......	......	......	......
2580.35	1	......	......	......	......	......
2593.71	1	......	......	......	......	......
(2)	4	2631.39	2631.38	......	2631.39	2631.370
2637.92	1	......	......	......	......	......
2640.89	2	......	......	......	......	......
2645.59	1	......	......	......	......	......
2649.47	0	......	......	......	......	......
2650.73	2	......	......	......	......	......
2658.29	0	......	......	......	2659.0	......
2672.43	0	......	......	......	2673.3	......
2675.25	2	......	......	......	......	......
......	......	......	......	......	2677.4	......
2682.42	2	......	......	......	......	......
2687.75	2	......	......	......	......	......
(*)	......	......	......	......	2689.8	......
2695.28	2	......	......	......	......	......
2697.20	1	......	......	......	......	......
2701.53	1	......	......	......	......	......
2716.25	1	......	......	......	......	......
2813.61	1	......	......	......	......	......
2831.40	2	......	......	......	......	......
2858.14	1	......	......	......	......	......
2866.28	1	......	......	......	......	......
2869.73	1	......	......	......	......	......
2873.10	1	......	......	......	......	......
2881.70	7	2881.70	2881.73	2881.7	2881.70	2881.690
2899.52	2	......	......	......	......	......
2904.01	2	......	......	......	......	......
2905.59	2	......	......	......	......	......
2924.04	2	......	......	......	......	......
2976.42	1	......	......	......	......	......
2987.99	2	2987.77	2987.77	2987.8	2987.77	2987.750
3002.37	1	......	......	......	......	......
3012.55	1	......	......	......	......	......
3034.37	0	......	......	......	......	......
3043.70	2	......	......	......	......	......
3086.44	5	......	3086.6	3087.2	......	3086.479
3093.28	4	......	3093.6	3094.5	......	3093.694
3096.92	3	......	......	......	......	......
3103.80	1	......	......	......	......	......
3106.14	1	......	......	......	......	......
3130.48	3	......	......	......	......	......
3147.01	1	......	......	......	......	......
3149.67	4	......	......	......	......	......

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
3165.78	4	......	......	......	......	......
3185.28	3	......	......	......	......	......
3188.85	1	......	......	......	......	......
......	......	......	......	......	3191.1	......
3192.69	1	......	......	......	......	......
3196.12	2	......	......	......	......	......
3199.42	2	......	......	......	......	......
3203.87	2	......	......	......	......	......
3210.27	3	......	......	......	......	......
3230.43	2	......	......	......	......	......
3234.05	3	......	......	......	......	......
3241.80	4	......	......	......	......	......
(1)	2	......	......	......	......	3247.684
3258.45	2	......	......	......	......	......
3270.33	1	......	......	......	......	......
3313.85	0	......	......	......	......	......
3333.48	1	......	......	......	......	......
......	......	......	......	......	......	3438.444
3464.14	2	......	......	......	......	......
3470.68	1	......	......	......	......	......
3481.55	2	......	......	......	......	......
3486.79	4	......	......	......	......	......
3532.02	0	......	......	......	......	......
3537.70	0	......	......	......	......	......
3548.24	1	......	......	......	......	......
3563.61	1	......	......	......	......	......
3570.05	1	......	......	......	......	......
3576.15	2	......	......	......	......	......
3590.77	3	......	3591.0	......	......	......
3702.01	2	......	......	......	......	......
3713.23	2	......	......	......	......	......
3762.42	2	......	......	......	......	......
3774.74	2	......	......	......	......	......
3791.13	3	......	3791.8	3791.5	3791.1	......
3796.18	4	......	3796.5	3796.0	3795.9	3796.364
3806.60	6	......	3806.90	3807.0	......	3806.802
(2)	1	......	......	......	3826.7	......
(2)	1	......	......	......	3834.4	......
3853.01	2	......	3854.02	......	3854.00	3853.812
3856.09	4	......	3856.19	3856.0	3856.20	3856.193
3862.51	4	......	3862.80	3862.5	3862.75	3862.743
3870.64	1	......	......	......	......	......
3891.55	2	......	......	......	......	......
3905.37	3	3905.66	3905.71	3905.5	......	3905.726
3924.44	3	......	......	......	......	......
3991.62	2	......	......	......	......	......

(1) This line apparently due to copper.
(2) Lines in these positions attributed to iron.
(*) A line here too faint to measure accurately.

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont	Lunt	Crookes
4016.28	1					
			4021.0			
			4030.1			
4058.49	1					
4088.88	6				4089.0	4089.016
4102.62	0	4103.09	4103.2			
			4103.7			
4116.15	6				4116.35	
4128.11	8		4128.1	4128.2	4128.20	4128.189
4130.96	10		4131.0	4131.3	4131.08	4131.192
4183.67	0					
4190.92	2		4191.1		4191.00	
4198.25	2				4198.43	
4212.66	1					
4236.45	1					
4277.95	1					
4314.32	1					
4328.40	1					
4338.57	2					
4372.33	1					
4377.80	1					
4494.02	1					
4552.50	20		4552.75	4552.5	4552.82	4552.841
4567.66	16		4567.95	4567.0	4567.82	4568.123
4574.66	12		4574.90	4574.5	4774.86	4574.823
4619.60	1					
4631.22	2					
4632.94	1					
4638.36	1					
4654.08	4					
4665.76	1					
4673.45	0					
4683.10	2					
4709.20	1					
4716.71	4					
4730.52	1					
			4764.20			
4776.58	1					
4800.43	1					
4813.28*	3					
4819.57*	4					
4828.84*	6					
4837.97	1					
4842.35	1					
4883.51	1					
4907.50	1					
4921.86	1					
4943.16	1					
5041.17	6		5043	5045.5	5042.4	5042.715
5056.10	8		5057.3	5060.0	5057.1	5057.832
5092.00	1					
5101.42	0					
5113.70	1					
5182.13	1					

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont	Lunt	Crookes
5185.64	1					
5193.21	1					
5196.62	1					
5202.85	2					
5219.05	1					
5240.63	0					
5295.66	0					
5417.36	1					
5438.58	1					
5456.61	1					
5469.18	1					
5472.90	1					
5494.82	0					
5576.23	0					
5589.18	0					
5593.82	0					
5632.72	0					
5639.13	1					
		5645.83				
5669.63	2	5665.78				
		5684.71				
5688.83	1	5690.65				
5694.68	0					
5701.26	1	5701.32				
5706.23	1	5708.62				
5716.08	1					
5739.20†	4					
		5772.36				
5785.45	0					
5800.25	1					
5806.30	1					
5845.06	1					
5867.33	2					
5914.7	1					
		5948.77		5948.?		
5957.80	2			5960.3	5960.3	5961.6
5979.20	2			5978.9	5981.3	5982.0
6329.71	1					
6339.39	2					
6347.01	6	6342.2	6347.1	6342.2	6346.9	6346.962
6363.10	1					
6371.09	4	6369.7	6371.4	6369.7	6372.2	6371.032
6662.3	1					
6671.2	1					

* Observed also by A. Fowler, "Silicon Lines in the Spectra of B-type Stars," Royal Astronomical Society, *Monthly Notices*, **76**, 196–197, 1916.

† Observed also by A. Fowler (*ibid.*).

a possible sixth. Several other frequency differences have been observed, but unless series can be found relating them, their significance may be questioned in many cases. For this reason they are omitted.

In conclusion it may be said that, so far as is known, the present work is the only extensive work on the spectrum of silicon which has been published in International units. The discovery of so large a

TABLE II

VACUUM-SPARK SPECTRUM OF SILICON

(A list of doubtful lines observed on one plate only)

Wave-Length in I.A. Units	Intensity	Wave-Length in I.A. Units	Intensity
2125.77	0	3420.32	0
2468.19	1	3423.66	0
2482.58	1	3430.51	0
2505.03	0	3499.61	1
2546.69	0	3651.66	2
2566.33	1	3661.97	0
2578.22	1	3670.49	1
2619.27	1	3681.38	1
2634.35	1	3779.47	1
2669.26	1	3837.65	1
2672.43	1	4141.04	0
2678.31	0	4304.15	0
2686.34	1	4321.81	0
2690.64	1	4324.40	0
2704.84	2	4598.21	0
2722.63	1	4599.62	0
2724.98	1	4624.91	0
2785.45	1	4627.35	0
2793.75	1	4657.20	0
2855.41	0	4659.17	0
2856.76	0	4900.48	0
2875.10	1	4949.72	0
2887.68	2	4958.19	0
2938.98	0	5105.98	0
2945.72	1	5108.94	0
2952.83	2	5206.66	0
2980.33	1	5392.75	0
2996.97	1	5448.46	0
3053.02	2	5451.91	0
3121.99	1	5466.70	0
3266.95	2	5540.16	0
3276.10	2	5622.36	0
3281.64	1	5796.71	0
3283.69	1	5813.50	0
3402.96	0	5827.16	0
3413.41	0	6079.23	0
3416.94	0	6099.73	0

number of new lines shows that for an element with which it is very difficult to get results when the usual light sources are used the vacuum-spark is a very powerful source.

PHYSICAL LABORATORY
UNIVERSITY OF MICHIGAN
January 1923

A SPECTROSCOPIC METHOD OF DERIVING THE PARALLAXES OF THE B-TYPE STARS[1]

By WALTER S. ADAMS and ALFRED H. JOY

ABSTRACT

Spectroscopic method of determining the absolute magnitudes of B-type stars.—The correlation between spectral type and absolute magnitude found to exist in the case of stars of the A-type of spectrum has been extended to those of the B-type. The spectrograms of 300 such stars have been classified as accurately as possible, according to the Harvard system, and *the character of the lines* has been indicated as nebulous or sharp. With the aid of absolute magnitudes derived (1) from trigonometric parallaxes, (2) from the moving clusters Pleiades, Perseus, Orion, and Scorpio-Centaurus, (3) from mean values obtained by Plummer, Charlier, Kapteyn and others, the correlation between type and absolute magnitude has been established for these stars. The agreement found in this way is satisfactory. The mean difference between spectroscopic absolute magnitudes and those for the four moving clusters is -0.2, and for the trigonometric values $+0.4$. The *principal uncertainties* arise in the case of the O-type stars and the early B-stars with sharp and diffuse lines.

The mean spectroscopic absolute magnitudes for 299 of the stars have been compared with the reduced proper motion $0.2\,m+\log\mu$, the stars being divided into 12 groups for this purpose. The curve found in this way is nearly a straight line except for the stars of extremely small proper motion. This result yields valuable evidence for the accuracy of the method and indicates that the reduced proper motion can be used to derive the absolute magnitudes of these stars with a considerable degree of precision.

In a recent publication[2] we showed that the individual parallaxes of the A-type stars could in most cases be determined with a satisfactory degree of accuracy from their spectral types. A close correlation was found to exist between absolute magnitude and the subdivision of the A-type of spectrum based upon the Harvard system of classification. It was stated that similar considerations could probably be applied to B-type stars and the purpose of this communication is to show the results of such an application to about 300 stars of this type for which we have obtained spectrograms at Mount Wilson.

The first step in this investigation, as in the case of the A-type stars, was an accurate determination of spectral type. The Harvard system was followed as closely as possible and use was made of spectrograms of typical stars for reference purposes. The results

[1] *Contributions from the Mount Wilson Observatory*, No. 262.

[2] *Mt. Wilson Contr.*, No. 244; *Astrophysical Journal*, **56**, 242, 1922.

show, in general, close accordance with the spectral types of the Revised Draper Catalogue. The principal differences come in types B5 and B8, and it is evident that, if the successive subdivisions from B0 to B9 are to be considered as equal steps, the interval from B5 to B8 is too large as compared with similar intervals preceding and following. Thus the difference in type between B2 and B5, or between B8 and A0 is greater than between B5 and B8. The Harvard observers have classified no stars in this interval and it seems probable that the spectra listed by them as B5 could be classed advantageously as B7 and the other subdivisions revised to correspond. We have in addition adopted provisionally in the case of the Harvard stars of type Oe5 the nomenclature suggested by H. H. Plaskett,[1] using the notation O5, O6, etc., to indicate the dark line stars immediately preceding B0. The spectra have further been characterized by the letters "n" and "s" to indicate nebulous or diffuse and sharp lines, respectively.

The correlation of these spectral types with absolute magnitude was first attempted through the use of trigonometric parallaxes and those derived from moving clusters such as the Pleiades, Orion, Perseus, and Scorpius. A satisfactory relationship was obtained in this way, but it seemed desirable to make use in addition of the important statistical investigations of Plummer,[2] Charlier,[3] Kapteyn,[4] and others on the mean magnitudes of the B-type stars. Accordingly, their values, which, as is well known, show clearly the decrease in absolute magnitude with advancing spectral type, were combined and assigned high weight in the construction of our reduction curves.

Since many of the trigonometric parallaxes of the B-type stars are negative, it is not possible to compute the absolute magnitudes of the individual stars in all cases. Accordingly, the procedure adopted has been to divide the stars into groups with a small range in spectral type and to compute the average absolute magnitude for each group on the assumption that the sum of the errors of the

[1] *Publications of the Dominion Astrophysical Observatory*, **1**, No. 30.

[2] *Monthly Notices of the Royal Astronomical Society*, **73**, 174, 1912.

[3] *Meddelanden från Lunds Observatorium*, Serie II, No. 14.

[4] *Mt. Wilson Contr.*, No. 147; *Astrophysical Journal*, **47**, 255, 1918.

trigonometric parallaxes for the stars in each group is zero. The formula used is one derived by Strömberg and has the form

$$M = 5 + 5 \log \Sigma\pi_0 - 5 \log \Sigma 10^{-0.2\,m}$$

where M is the required absolute magnitude, π_0 the observed trigonometric parallax, and m the apparent magnitude.

The final values upon which our curves are based are as follows:

MOVING CLUSTERS AND TRIGONOMETRIC PARALLAXES

B0.5n	−2.8;	B1.8n	−1.4;	B3.3	−0.6;	B5.3n	−0.1;	B8.2n	+0.3
		B1.7s	−1.9					B8.4s	−0.1

MEAN ABSOLUTE MAGNITUDES FROM STATISTICAL INVESTIGATIONS[1]

O5 to O9	−1.8;	B1	−2.4;	B4	−0.9;	B8.5	0.0;	A0	+0.4;	A2	+1.3

In addition to these values those furnished by the curves for the A-type stars are available for stars of type B8 or later. These are in good agreement with the results of Plummer and have been adopted directly.

Two outstanding difficulties are the absolute magnitudes of the O-type stars and the question of the difference in absolute magnitude between stars of the early types with sharp and diffuse lines. The material is insufficient for a definite answer to either. The investigation of Gyllenberg[2] on stars of Harvard type Oe5 would indicate that their average absolute magnitude is considerably fainter than the probable value for B0 and B1 stars. Accordingly, we have made a turning point in our curves at B0 which would bring them into agreement with Gyllenberg's result at about O8. The values for these stars must, however, necessarily be uncertain.

Our investigation of the A-type stars showed that the average absolute magnitude of those showing sharp lines was about 0.7 magnitude brighter than that of stars with diffuse lines. This applies to spectra as early as B8. In the case of stars of types

[1] These include the results of Plummer, Charlier, Kapteyn, Dyson, Gyllenberg, and Malmquist.

[2] *Arkiv for Matematik, Astronomi och Fysik, K. Svenska Vetenskapsakademien*, **16**, No. 24.

Bo to B5, however, the question is very doubtful on account of the small amount of material available for study. There is some indication from a number of stars in Orion of average type about B2 that the stars having sharp lines are somewhat brighter than those with diffuse lines. We have made a partial compromise by drawing two curves, one for stars of each class of lines, which come together at Bo. The procedure is somewhat arbitrary, but probably no very serious error is introduced since the number of stars with sharp lines between types Bo and B5 is small. It is quite possible that some of the stars in Orion which have sharp spectral lines and great intrinsic brightness, such as γ Orionis, may resemble the c-class of stars of high luminosity to which β Orionis of type B8 belongs. In the absence of other data, however, they have been included in our list.

The values resulting from our reduction curves are as follows:

TABLE I

Spectrum	Diffuse	Sharp
O5–O9	$-2^{M}5$	$-2^{M}5$
Bo	−3.1	−3.1
B1	−2.4	−2.6
B2	−1.5	−2.0
B3	−0.9	−1.5
B4	−0.6	−1.2
B5	−0.5	−1.1
B6	−0.3	−0.9
B7	−0.1	−0.8
B8	+0.1	−0.6
B9	+0.5	−0.2
Ao	+0.9	+0.2
A1	+1.3	+0.6
A2	+1.7	+0.9

These values have been used in the determination of the absolute magnitudes of 300 stars of the O and B types for which we have spectrograms at Mount Wilson. The list is given in Table II. The c-stars have been included, although it may prove that as in other types these stars require separate treatment.

TABLE II

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
5......	5.5	0".010	B9n	B8	+0.5	+0".010	
18......	5.5	.034	B9n	B8	+0.5	.010	
52......	6.0	.028	B9s	B9	−0.2	.006	−0.022 t
67......	5.4	.017	B5n	B3	−0.6	.006	
68......	5.4	.016	B8n	B9	+0.3	.010	
118......	5.1	.021	B8n	B5	0.0	.010	
124......	5.9	.028	B3s	B3	−1.5	.003	
159......	5.4	.022	B9s	B8	−0.2	.008	
169......	5.4	.014	B9n	B9	+0.5	.010	
224......	5.5	.039	B9s	B9	−0.2	.007	
263......	5.5	.038	B9s	B9	−0.2	.007	+0.010 g
265......	5.5	.027	B8n	B8	+0.3	.009	
284......	5.8	.028	B9s	B8	−0.2	.006	
340......	5.8	.025	B9n	B9	+0.5	.009	+0.017 t
347......	6.2	.011	B9n	B9	+0.5	.007	
379......	5.4	.053	B9s	B8	−0.2	.008	+0.013 g
412......	5.5	.035	B3n	B3	−1.2	.005	+0.009 g
419......	3.4	.043	B5n	B3	−0.5	.017	−0.001 t +0.011 g
425......	5.0	.010	B8s	B8	−0.6	.008	
457......	5.6	.010	cB6s	B5p	−0.8	.005	
459......	5.0	.041	B9n	B8	+0.2	.011	
488......	6.4	.013	B7s	B3p	−0.8	.004	
507......	6.4	.015	B2s	B1p	−2.0	.002	
519......	6.7	.013	cB3s	B0	−1.6	.002	
529......	6.1	.005	B5s	B5	−1.0	.004	
535......	7.0	.009	B8s	B8p	−0.6	.003	
544......	6.2	.012	B1n	B2	−2.8	.002	
546......	5.5	.019	B8n	B5	+0.2	.009	
641......	5.3	.022	B6n	B5	−0.6	.007	
648......	5.5	.045	B5n	B5	−0.4	.007	
666......	5.3	.054	A0n	A2	+0.7	.012	
678......	5.4	.043	B8n	B5	0.0	.008	+0.010 g
682......	5.5	.017	B9n	B9	+0.5	.010	
692......	5.6	.016	B8s	B5	−0.6	.006	
699......	5.8	.006	B9s	B9	−0.2	.006	
715......	5.6	.038	B9n	B9	+0.5	.010	+0.009 g
731......	5.5	.039	A1s	A0	+0.8	.011	
740......	5.4	.038	B7n	B5	0.0	.008	+0.009 g
742......	5.3	.045	B4n	B3	−0.8	.006	+0.011 g
744......	5.1	.041	B4n	B3	−0.6	.007	+0.010 g
767......	5.3	.036	B5n	B3	−0.4	.007	+0.009 g
780......	4.9	.032	B4s	B5	−1.0	.007	+0.008 g
783......	5.6	.049	B6n	B5	−0.2	.007	+0.011 g
790......	4.7	.044	B3n	B5	−0.8	.008	+0.010 g
791......	5.3	.033	A0n	A2	+0.9	.013	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
801	5.1	0″.021	B9n	A0	+0.5	+0″.012	
802	5.5	.040	B8s	B8	−0.4	.007	+0.010 g
807	5.8	.011	B1n	B3	−2.4	.002	
833	5.6	.032	B8n	B5	0.0	.008	+0.008 g
838	3.1	.046	B5n	B5	−0.4	.020	+0.006 t +0.011 g
839	5.0	.015	B1s	B2	−2.3	.003	
841	5.5	.019	B8n	B8	+0.3	.009	
852	3.8	.053	B8n	B8	+0.2	.019	+0.014 g
855	5.6	.057	B9n	B8	+0.5	.010	
856	4.4	.049	B7n	B5	−0.1	.013	+0.013 g
857	5.1	.017	B9n	B8	+0.5	.012	
860	4.0	.054	B9s	B5	−0.2	.014	+0.014 g
861	5.8	.044	B9n	B8	+0.3	.008	+0.011 g
865	4.2	.059	B5n	B5	−0.4	.012	+0.015 g
869	3.0	.052	B8n	B5p	+0.3	.029	+0.013 g
872	5.5	.059	B8n	B8	0.0	.008	
877	3.8	.053	B9n	B8	+0.5	.022	+0.014 g
879	5.2	.053	B8n	B8p	+0.1	.010	+0.013 g
886	5.7	.011	B3n	B3	−0.9	.005	
893	5.5	.012	B7n	B8	−0.1	.008	
896	4.9	.005	B9n	B9	+0.5	.013	
898	5.3	.039	B4n	B5	−0.6	.007	+0.010 g
904	5.5	.020	B2n	B3	−1.2	.005	
910	3.0	.039	B1n	B1	−2.0	.010	−0.008 t
913	4.0	.018	O8n	Oe5	−2.4	.005	−0.004 t
926	5.2	.024	B4n	B5	−0.8	.006	
956	5.4	.013	B5s	B3	−1.0	.005	
992	5.5	.014	B4n	B3	−0.7	.006	
1015	5.4	.041	B9n	B9	+0.5	.010	
1039	5.4	.019	B6n	B6	−0.4	.007	
1070	5.7	.012	B9n	B9	0.0	.007	
1084	5.3	.018	B7n	B5	−0.4	.007	
1097	5.4	.027	B8s	B9	−0.7	.006	
1107	4.3	.023	B4n	B5	−0.8	.010	+0.018 t
1139	4.4	.009	B0s	B0	−3.0	.004	−0.026 t
1147	3.8	.007	B1s	B3	−2.3	.006	0.000 t +0.007 g
1159	3.9	.004	B3s	B3	−1.5	.008	+0.003 t +0.007 g
1163	4.7	.145	A0n	A0	+0.9	.017	
1165	5.7	.024	B9n	B8	+0.5	.009	
1177	5.6	.067	A0n	B9	+1.1	.013	
1204	3.3	.081	B3s	B3	−1.5	.011	+0.017 t
1216	5.5	.011	B3s	B3	−1.5	.004	
1249	5.8	.029	O9s	B0p	−3.0	.002	
1262	3.7	.017	B7s	B5	−0.8	.013	+0.007 g
1274	5.1	.045	B5n	B3	−0.4	.008	+0.011 g

TABLE II—*Continued*

Boss No.	*m*	*μ*	Spectrum M.W.	Spectrum H.D.	*M*	Spec. *π*	Group or Trig. *π*
1283	5.6	0".010	B3n	B3	−0.9	+0".005	
1284	4.6	.005	B2s	B3	−2.0	.005	+0.007 g
1293	5.9	.015	B9n	B9	+0.7	.009	
1295	5.6	.015	B2s	B3	−2.0	.003	+0.007 g
·1301	3.4	.006	B1s	B1	−2.3	.007	+0.007 g
1303	1.7	.020	B2s	B2	−2.0	.018	+0.021 t +0.007 g
1304	1.8	.180	B9s	B8	−0.2	.040	−0.002 t
1307	6.2	.009	B3n	B3	−0.8	.004	
1310	5.7	.019	B9s	B9	0.0	.007	
1314	4.7	.014	B3n	B2	−0.9	.008	−0.015 t +0.007 g
1318	5.5	.031	B9n	A0	+0.5	.010	
1320	5.4	.039	B9n	A0	+0.5	.010	
1328	5.7	.013	B4n	B3	−0.8	.005	
1332	5.5	.009	B2s	B3	−2.0	.003	+0.007 g
1339	2.5	.004	B1n	B0	−2.4	.010	−0.018 t +0.007 g
1339C	6.9	.003	B6n	....	−0.4	.003	
1340	4.6	.012	B1s	B3	−2.6	.004	+0.007 g
1346	5.6	.006	B4n	B3	−0.6	.006	
1349	5.4	.007	B3n	B2	−0.9	.005	
1354	5.3	.032	B3n	B3	−0.9	.006	
1357A	3.7	.011	O8n	Oe5	−2.1	.006	
1357B	5.6	.011	B2s	....	−1.8	.003	
1361	5.6	.004	B2s	B1	−1.8	.003	+0.007 g
1363	5.4	.005	O8n	Oe5	−2.1	.003	−0.016 t
1364	4.6	.002	B2s	B3	−1.4	.006	
1365	5.2	.020	B6n	B1	−0.3	.008	
1366A	2.9	.005	O9n	Oe5	−2.4	.009	
1366B	7.3	.005	B9s	....	−0.2	.003	
1370	1.8	.002	B1n	B0	−2.4	.014	+0.008 t +0.007 g
1375	3.0	.028	B4n	B3p	−0.6	.019	
1382	5.8	.024	B3n	B1	−1.0	.004	
1389	3.8	.001	O9s	B0	−2.8	.005	+0.007 g
1398	2.0	.011	B0n	B0	−2.8	.011	−0.020 t +0.007 g
1398B	4.2	.011	B1n	....	−2.4	.005	
1399	5.0	.014	B4n	B3	−0.8	.007	+0.007 g
1423	5.7	.059	B9n	A2	+0.3	.006	
1435	2.2	.006	B1n	B0	−2.4	.012	+0.029 t +0.007 g
1507	4.7	.015	cB2s	cB2p	−2.3	.004	
1517	5.6	.123	B4n	B3	−0.8	.005	+0.074 t
1523	5.4	.007	B5s	B3	−1.0	.005	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
1567	5.8	0".041	B2s	B2	−2.0	+0".003	
1568	5.3	.021	B8n	B9	+0.3	.010	
1572	5.4	.017	B9s	B9	−0.2	.008	
1578	6.3	.013	B4s	B2	−1.0	.003	
1609	2.0	.007	B1s	B1	−2.3	.014	+0.012 t
1706	4.7	.008	O8n	Oe5	−2.4	.004	−0.004 t
1739	5.2	.012	B8n	B8	+0.1	.010	
1751	5.7	.023	B8s	B8	−0.4	.006	
1754	5.3	.019	B6n	B5	−0.2	.008	
1807	5.8	.031	B3s	B3	−1.8	.003	
1817	3.1	.009	cB1s	cB5p	−2.8	.007	
1899	4.9	.016	O9s	Oe	−2.8	.003	
1905	6.5	.029	B9n	B8	+0.5	.006	+0.006 t
1906	5.6	.042	B8n	B8	+0.1	.008	+0.012 t
1944	3.1	.066	B9n	B8	+0.5	.030	+0.022 t
1955	6.0	.042	B3n	B3	−0.8	.004	
1998	5.7	.016	B2s	B3	−2.0	.003	
2019	5.8	.024	B3n	B5	−0.9	.005	
2045	5.3	.027	B8n	A0	0.0	.009	
2159	5.5	.014	B3s	B3	−1.5	.004	
2353	5.7	.098	A0n	A0	+0.7	.010	
2451	5.5	.032	B9n	B8	+0.5	.010	
2465	5.3	.052	A1s	B8	+0.6	.011	
2492	5.5	.040	B9n	B9	+0.7	.011	
2600	5.0	.031	B4n	B3	−0.6	.008	
2698	1.3	.247	B9n	B8	+0.5	.069	+0.055 t
2712	6.1	.021	A0n	A0	+0.9	.009	
2724	5.4	.074	A0n	A0	+0.9	.013	
2748	6.5	.008	B2n	B3	−1.2	.003	
2788	5.2	.052	B9n	B9	+0.5	.011	
2792	5.0	.054	B5s	B5	−1.1	.006	
2797	5.5	.048	B8n	B9	+0.1	.008	
2798	5.8	.015	B9n	B9	+0.5	.009	
2804	3.8	.009	B1s	B0p	−2.3	.006	+0.039 t
2839	6.9	.013	B9n	A2	+0.5	.005	
2866	5.4	.049	B8n	B9	+0.3	.010	
3045	5.8	.011	B3n	B3	−0.8	.005	
3055	4.8	.061	B9n	B9	+0.5	.014	
3138	5.3	.020	B3n	B3	−0.9	.006	
3190	3.4	.110	A0n	A2	+0.9	.032	
3309	5.0	.135	B9n	A0	+0.5	.013	
3329	6.0	.054	A0n	A3	+0.7	.009	
3392	5.1	.027	B9n	B9	+0.7	.013	
3428	6.3	.039	A0n	A0	+0.7	.008	
3476	1.2	.055	B3n	B2	−0.9	.038	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
3546	6.2	0″.053	A0n	A0	+0.9	+0″.009	
3604	5.2	.064	B6n	B8	−0.4	.008	
3653	5.6	.044	B9s	B9	−0.4	.006	
3724	2.6	.049	B3n	B3p	−1.0	.019	+0.013 g
3756	5.8	.026	A0n	B9	+0.9	.010	
3820	6.4	.029	A0n	A0	+0.7	.007	
3875	6.3	.029	B9s	B9	−0.2	.005	
3915	5.4	.051	B9n	B9	+0.5	.010	
3944	5.6	.028	B4n	B3	−0.8	.005	
3946	6.1	.016	B9n	B9	+0.5	.008	
3955	5.2	.028	B6s	B5	−0.8	.006	
3988	5.1	.014	B6n	B8	−0.2	.009	
4008	5.4	.055	B5n	B8	−0.4	.007	
4019	4.8	.042	B4n	B3	−0.6	.008	+0.011 g
4028	5.2	.099	A0n	A2	+0.9	.014	
4033	5.1	.037	B4n	B3	−0.8	.007	+0.010 g
4034	4.7	.034	B3n	B3	−0.8	.008	+0.009 g
4037	5.4	.046	B6n	B5	−0.2	.008	+0.012 g
4038	5.4	.047	B4n	B3	−0.8	.006	
4041	5.9	.034	B9n	B8	+0.5	.008	
4043	5.9	.046	B5n	B5	−0.4	.005	+0.012 g
4058	5.4	.052	B6n	B8	−0.2	.008	
4086	2.9	.031	B0n	B1	−2.8	.007	−0.019 t +0.008 g
4087C	5.1	.034	B3n	....	−1.2	.005	+0.009 g
4097	5.6	.065	B9n	A0	+0.5	.010	
4115	4.7	.047	B2n	B3	−1.2	.007	+0.012 g
4117	4.3	.034	B2n	B3	−1.6	.007	+0.015 t
4151	5.5	.009	B9n	B8	+0.5	.010	
4178	5.2	.024	B4n	B3	−0.8	.006	+0.006 g
4184	5.5	.026	A0n	A3	+0.9	.012	
4198	4.9	.028	B3n	B3	−0.9	.007	+0.008 g
4213	5.0	.042	B9n	cB8p	+0.7	.014	
4291	6.4	.011	A1n	A0p	+1.3	.010	
4338	6.2	.038	B9n	B3	+0.5	.007	
4345	5.6	.008	B3n	B3	−0.9	.005	
4368	3.2	.024	B5s	B5	−1.1	.014	
4427	5.4	.016	B5n	B5	−0.5	.007	
4442	5.7	.108	A0n	A2	+0.9	.011	
4468	5.7	.023	B9n	A0	+0.5	.009	
4527	5.7	.006	B2n	B3	−1.5	.004	
4548	3.9	.014	B2s	cB5p	−2.0	.007	
4552	4.4	.026	B9n	A2	+0.5	.017	
4562	5.1	.018	B3s	B3	−1.5	.005	+0.002 t
4573	5.8	.026	B6n	B8	−0.2	.006	
4576	6.2	.014	B0s	B1	−2.8	.002	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
4612......	5.4	0".005	B0n	cB0	−2.6	+0".003	
4613......	6.0	.010	B1n	B1	−2.8	.002	
4620......	5.4	.009	B6n	B5	−0.2	.008	
4640......	6.5	.010	B9n	B8	+0.5	.006	
4668......	6.0	.007	B8n	B8	0.0	.006	
4685......	5.2	.035	A0n	A0	+0.9	.014	
4687......	5.8	.031	B3s	B3	−1.5	.003	
4702......	5.4	.007	B9s	B8	−0.2	.008	
4721......	5.5	.024	B8s	B8	−0.4	.007	
4740......	5.0	.022	B8n	B5	+0.2	.011	
4772......	5.8	.014	B3n	B2	−0.9	.005	
4777......	7.8	.020	B8n	B3	0.0	.003	
4783......	5.0	.024	B6s	B5	−1.0	.006	
4794......	5.5	.011	B3n	B3	−0.9	.005	
4813......	5.6	.018	B8n	B8	+0.3	.009	
4816......	5.4	.025	B7s	B5	−0.8	.006	
4821......	5.8	.002	B6s	B8	−0.6	.005	
4842......	5.2	.025	B3n	B3	−1.2	.005	
4873......	5.1	.009	B6n	B5	−0.3	.008	
4883......	5.4	.017	B3n	B3	−0.9	.005	
4917......	5.4	.013	B4n	B	−0.6	.006	
4936......	4.1	.129	B9n	B8	+0.5	.019	
4942......	4.9	.015	B8s	B5	−0.4	.009	
4954......	5.7	.023	B8n	B8	+0.1	.008	
4974......	5.8	.037	A0n	A	+0.9	.010	
4987......	5.4	.014	B8n	B9	+0.3	.010	+0.006 t
5003......	5.0	.004	B4n	B	−0.7	.007	
5073......	6.3	.007	O8n	Oe5	−2.6	.002	
5083......	5.5	.015	B0n	....	−2.7	.002	+0.009 t
5087......	5.8	.022	B5n	B3	−0.4	.006	
5088......	6.5	.029	B8n	B3	+0.1	.005	
5102......	4.9	.012	B5s	B3	−1.1	.006	
5105......	4.8	.051	A0n	A3	+0.9	.017	
5113......	5.4	.024	B5n	B3	−0.5	.007	
5122......	5.4	.031	B9n	B8	+0.5	.010	
5130......	6.0	.020	B7n	B5	−0.1	.006	
5156......	5.1	.008	B4n	B3	−0.7	.007	
5160......	6.2	.038	B9n	A	+0.5	.007	
5188......	5.0	.094	A0n	A	+1.1	.017	
5210......	5.8	.011	B2n	....	−1.9	.003	+0.008 t
5240......	5.2	.016	B8s	B8	−0.6	.007	
5307......	5.9	.025	B4n	B3	−0.7	.005	
5316......	6.5	.009	B4n	B5	−0.6	.004	
5322......	5.6	.017	B8s	A	−0.6	.006	
5325......	5.4	.006	B3s	B3	−1.2	.005	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
5389	5.8	0".005	cB8s	cB8p	−0.6	+0".005	
5393	4.0	.025	B9n	A	+0.5	.020	
5405	5.5	.018	B8n	B8	+0.1	.008	
5414	5.2	.004	B5n	B3	−0.4	.008	
5465	5.4	.015	B9s	B8	−0.2	.008	
5474	5.0	.028	O9n	Oe5	−2.4	.003	
5512	5.8	.011	B0n	B	−3.0	.002	
5516	5.2	.004	B3n	B3	−0.8	.006	
5525	5.4	.018	B8n	B5	+0.1	.009	
5532	3.3	.012	B1s	B1	−2.3	.008	−0.011 t
5550	6.3	.026	A0n	A	+0.9	.008	
5573	6.0	.052	A1n	A	+1.3	.011	
5585	5.6	.038	B9n	A	+0.7	.009	
5627	5.0	.001	B3n	B3	−0.8	.007	
5629	5.5	.015	B6n	B3	−0.2	.007	
5641	5.6	.036	B9n	A2	+0.5	.010	
5653	6.0	.006	B4s	B5	−1.2	.004	
5681	5.6	.036	B9n	A	+0.5	.010	
5687	5.2	.011	O9n	Oe5	−2.8	.003	
5706	5.4	.026	B7s	B5	−0.8	.006	
5738	6.2	.028	B9n	A	+0.3	.007	
5757	5.4	.019	B6s	B5	−1.0	.005	
5796	5.7	.011	B0n	....	−3.1	.002	
5810	4.5	.013	B3s	B3	−1.2	.007	
5856	5.2	.021	B2n	B2	−1.8	.004	
5879	5.7	.037	A0n	B9	+1.1	.012	
5933	3.6	.032	B7n	B5	0.0	.019	+0.004 t
5967	5.3	.024	B8n	B8	+0.3	.010	
5973	5.2	.017	B9n	A3	+0.5	.011	
6075	5.3	.016	B9n	B9	+0.5	.011	
6128	5.4	.033	A1n	A	+1.5	.017	
6142	6.0	.006	B0n	Bp	−3.0	.002	+0.030 t
W B 3^h 147	6.0	....	B9n	B9	+0.5	.008	+0.038 t
Bond 708	6.3	.020	B2s	B1	−1.6	.003	
4 Gemin.	6.7	.014	B9n	B9	+0.5	.006	+0.005 t

A comparison of the results given by this method of reduction with those derived from moving clusters and from trigonometric parallaxes shows a satisfactory degree of accordance. Using the values of Rasmuson for moving clusters, except in the case of the

Orion cluster where a parallax of $+0''.007$ has been assumed, we find the following results for the absolute magnitudes:

	No.	Cluster	Spec.
Pleiades	8	$-0^{M}.1$	$+0^{M}.1$
Perseus	20	0.0	-0.5
Orion	17	-2.0	-2.0
Scorpio-Centaurus	11	-0.5	-1.0

The very remarkable case of Voûte's star of large proper motion and parallax, Boss 1517, enters into the results for trigonometric parallaxes. The spectrum is B4n. The comparison for stars with positive trigonometric parallaxes, including and excluding this star, is as follows:

No.	Trig.	Spec.
24	$-0^{M}.2$	$-1^{M}.0$
23	-0.4	-1.0

The difference is that to be expected from the effect of selection of the stars with positive trigonometric parallaxes, for which alone absolute magnitudes may be computed. This necessarily involves a selection of the positive errors in the determinations. A better comparison would be furnished by the use of the formula already employed in which the sum of the errors of the trigonometric parallaxes is assumed to be zero. For 34 stars with trigonometric parallaxes the mean absolute magnitude derived in this way is -1.4, as compared with the spectroscopic value of -1.0. It is of interest to note that of the trigonometric parallaxes (absolute), 24 are positive, 9 negative, and one is zero. The simple mean of these trigonometric parallaxes is $+0''.011$ and that of the spectroscopic parallaxes is $+0''.012$.

The catalogue which is given in Table II contains our results for the absolute magnitudes and parallaxes of stars of types O8 to B9 with the addition of a very few A-type stars. The successive columns give the Boss number, visual apparent magnitude, total proper motion, mean spectral type from the determinations of Adams and Joy, spectral type according to the Revised Draper Catalogue, absolute magnitude and parallax derived by the spectroscopic method, and values of the parallax as obtained from moving clusters or trigonometric determinations. The absolute magnitudes have been computed separately for the types of Adams and Joy

and then combined; in some cases, accordingly, there will be slight differences from those furnished directly by the mean spectral types. For stars of right ascension later than 18 hours the Harvard spectral types are from the earlier Harvard publications.

A valuable check on the accuracy of the absolute magnitudes derived by this method is furnished by the relationship of M to total proper motion. For such a comparison, especially when a wide range in apparent magnitude is involved, the best procedure is doubtless to use the reduced proper motion as has been done in the case of other stars by Lundmark and Luyten.[1] For each star the value of $0.2\,m+\log\mu$ has been computed, and the mean value of M has been derived for groups of stars within narrow limits of this quantity. For very small values of the proper motion the uncertainty is, of course, considerable.

TABLE III

No.	$0.2\,m+\log\mu$		Mean M
	Limits	Mean	
12	<8.50	8.19	−2.01
11	8.50–8.74	8.65	−1.49
23	8.75–8.99	8.90	−1.37
23	9.00–9.14	9.07	−1.18
43	9.15–9.29	9.22	−0.80
38	9.30–9.39	9.34	−0.47
27	9.40–9.49	9.45	−0.48
32	9.50–9.59	9.55	−0.25
32	9.60–9.69	9.65	−0.15
29	9.70–9.79	9.74	+0.13
18	9.80–9.94	9.86	+0.21
11	>9.95	10.07	+0.61
299			

The results are shown graphically in Figure 1. The curve is very nearly a straight line except for the stars having extremely small proper motions.

The same conclusion may, therefore, be drawn for the B-type stars as for the A-type stars discussed previously, that the dispersion in absolute magnitude is small for stars of the same spectral subdivision with the same character of spectral lines. Occasional exceptions no doubt occur, as illustrated by β Orionis and such a star as Boss 1517, which is characterized by a large proper motion and

[1] *Lick Observatory Bulletin*, No. 339.

high radial velocity. Whether the B-type stars with bright lines are also exceptional in their behavior is less certain; but, since the reduction curves are based entirely upon the stars showing absorption lines, it has seemed preferable not to include material which is not of a homogeneous character.

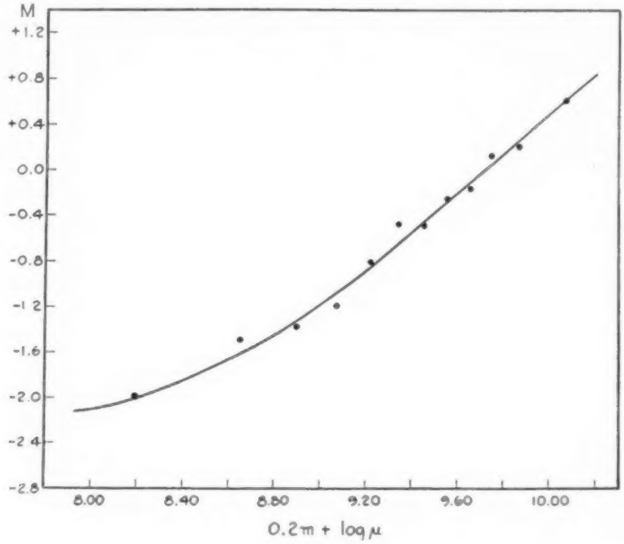


FIG. 1

An interesting publication by D. L. Edwards on "Spectroscopic Parallaxes of the Hotter Stars" has appeared while this investigation was in progress. The method used is to measure the relative intensities of the helium lines λλ 4144 and 4388 against the hydrogen lines Hδ and Hγ, respectively, and to determine the absolute magnitude from the correlation found to exist between these two quantities. To some extent this is a correlation with spectral type and in the case of the stars listed by Edwards, comparison of types with absolute magnitudes derived from the parallaxes which he uses does, in fact, show the existence of such a relationship. The successful development of this method should be of great value in showing the dispersion in absolute magnitude for stars of the same spectral subdivision.

MOUNT WILSON OBSERVATORY
March 1923

NOTICE TO CONTRIBUTORS

There is occasionally published in the *Astrophysical Journal* a Standing Notice (for instance, on pages 179–80 of the number for September 1917). This is principally intended to guide contributors regarding the manuscript, illustrations, and reprints. This notice contains the following paragraph:

> Where unusual expense is involved in the publication of an article, on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents, according to a uniform system.

The present sheet has been printed for amplifying further that paragraph.

The "uniform system" according to which "arrangements are made" is as follows: The cost of composition in excess of \$50, and of stock, presswork, and binding of pages in excess of 40 pages, for any one article shall be paid by the author or by the institution which he represents at the current rates of the University of Chicago Press. When four articles from one institution or author have appeared in any one volume, on which the cost of composition has amounted to \$50 each, or when the total cost of composition incurred by the *Astrophysical Journal* on articles for one institution has reached the sum of \$200, the entire cost of the composition, stock, presswork, and binding of any additional articles appearing in that volume shall be paid by the author or by the institution which he represents.

As to illustrations, the arrangement cannot be quite as specific, but it may be generally assumed that not more than three half-tone inserts can be allowed without payment by the author. The cost of paper, presswork, and binding for each full-page insert is about \$8.00, aside from the cost of the half-tone itself. In the matter of zinc etchings, considerable latitude has to be allowed, as in many cases diagrams take the place of more expensive tables. It may be assumed, however, that it will seldom be possible for the *Journal* to bear an expense of over \$25 for diagrams and text illustrations in any one article.

Contributors should notice that since January, 1917, it has been impossible to supply any free reprints of articles.

Reprints of articles, with or without covers, will be supplied to authors at cost. No reprints can be furnished unless a request for them is received before the *Journal* goes to press.

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. For details concerning the preparation of abstracts, see page 231 in the April, 1920, number of the *Journal*.

THE EDITORS

INDEX TO VOLUME LVII

SUBJECTS

INDEX TO VOLUME LVII

AUTHORS